Leaving Certificate Examination

Sample Paper 5

Applied Mathematics

Higher Level
2 hours and 30 minutes

400 marks

Examination Number

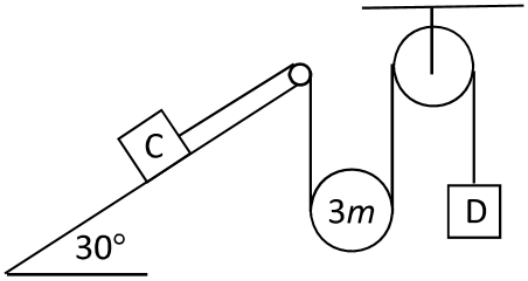
For examiner	
Question	Mark
1	/50
2	/50
3	/50
4	/50
5	/50
6	/50
7	/50
8	/50
~~9~~	~~/50~~
~~10~~	~~/50~~
Written Total	/400
Project	/100
Overall Total	/500
Overall Grade	

Sample Paper 5

Question 1

(a)

A particle C of mass $2m$ rests on a rough plane which is inclined at 30° to the horizontal.
The coefficient of friction between C and the plane is $\frac{\sqrt{3}}{21}$. A light inextensible string which passes under a smooth movable pulley of mass $3m$ connects C to a particle D of mass m, as shown in the diagram.



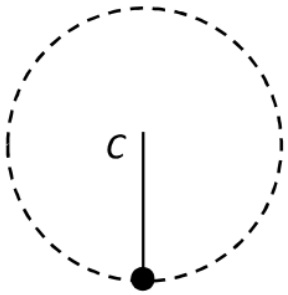
The system is released from rest. C moves up the plane.

(i) Show, on separate diagrams, the forces acting on the moveable pulley and on each of the masses.

(ii) Find in terms of m the tension in the string.

(b)

A particle is attached to one end of a light inextensible string of length 0.5 m. The other end of the string is attached to a fixed point *C*. The particle moves in a vertical circle.



The greatest and least tensions in the string are $3T$ and T, respectively.

Find the speed of the particle at the lowest point.

Question 2

(a)

A ball *E* is thrown vertically upwards with a speed of 42 m s^{-1}.

T (< 8) seconds later another ball, *F*, is thrown vertically upwards from the same point with the same initial speed.

(i) Find where ball *E* is after 5 s and the total distance it has travelled in this time.

(ii) Prove that when *E* and *F* collide, they will each be travelling with speed $\frac{1}{2}gT$.

(b)

The rate of decay at any instant of a radioactive substance is proportional to the amount of the substance remaining at that instant. The initial amount of the radioactive substance is N and the amount remaining after time t (hours) is x.

(i) Prove that $x = Ne^{-kt}$, where k is a constant.

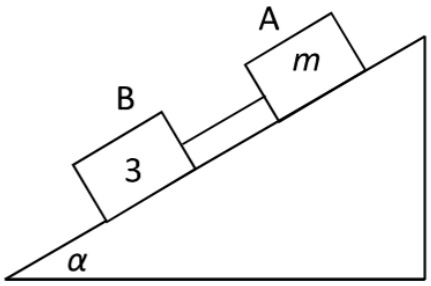
(ii) If the initial amount N was reduced to $\frac{N}{3}$ in 14 hours, find the value of k.

(iii) If the amount remaining is reduced from $\frac{N}{3}$ to $\frac{N}{4}$ in t hours, find the value of t.

Question 3

(a)

A block A of mass m is connected by a light inextensible string to a second block B of mass 3 kg.
They slide down a rough inclined plane which makes an angle α with the horizontal where $\tan\alpha = \frac{3}{4}$.
The string remains taut in the subsequent motion.
The coefficient of friction between A and the plane is $\frac{3}{4}$.
The coefficient of friction between B and the plane is $\frac{1}{3}$.
The system is released from rest.



Find

(i) the acceleration of B, in terms of m

(ii) the value of m if the tension in the string is 3·92 N.

(b)

Solve the difference equation $u_{n+2} - 7u_{n+1} + 10u_n = 4n^2 - 10n - 3$ given that $u_0 = 2$ and $u_1 = 8$.

Question 4

(a)

(i) Solve the differential equation

$$(1 + t^2)\frac{dr}{dt} = 1$$

given that $r = 0$ when $t = \frac{\pi}{4}$.

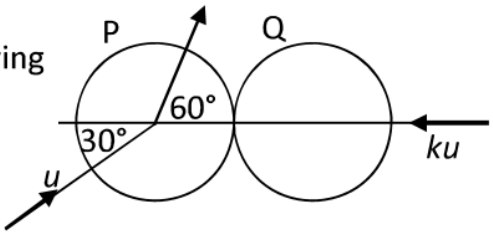
(ii) If

$$\frac{dy}{dx} = (y + 4)\cos^2 3x$$

and $y = -3$ when $x = 0$, find the value of y when $x = \frac{\pi}{6}$.

(b)

A smooth sphere P has mass $2m$ and speed u. It collides obliquely with a smooth sphere Q of mass m which is moving with speed ku, as shown in the diagram.
Before the collision, the direction of P makes an angle of 30° to the line of centres. After the collision, the direction of P makes an angle of 60° to the line of centres.



The coefficient of restitution between the spheres is e.

(i) Show that $k = \frac{\sqrt{3}(1-e)}{2(1+e)}$.

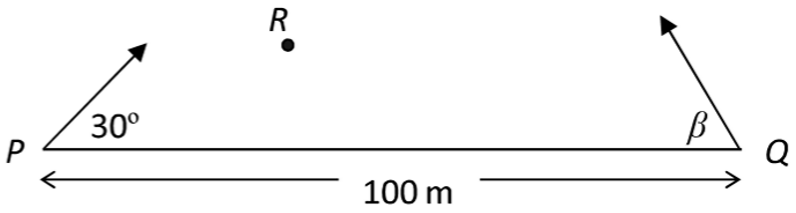
(ii) Find the speed of Q immediately after the collision.

Question 5

(a)

A particle is projected from a point P with speed 60 m s^{-1} at an angle of 30° to the horizontal. At the same time a second particle is projected from a point Q with speed 50 m s^{-1} at an angle β to the horizontal. P and Q are on the same horizontal level and are 100 m apart. The particles collide at R as shown in the diagram.

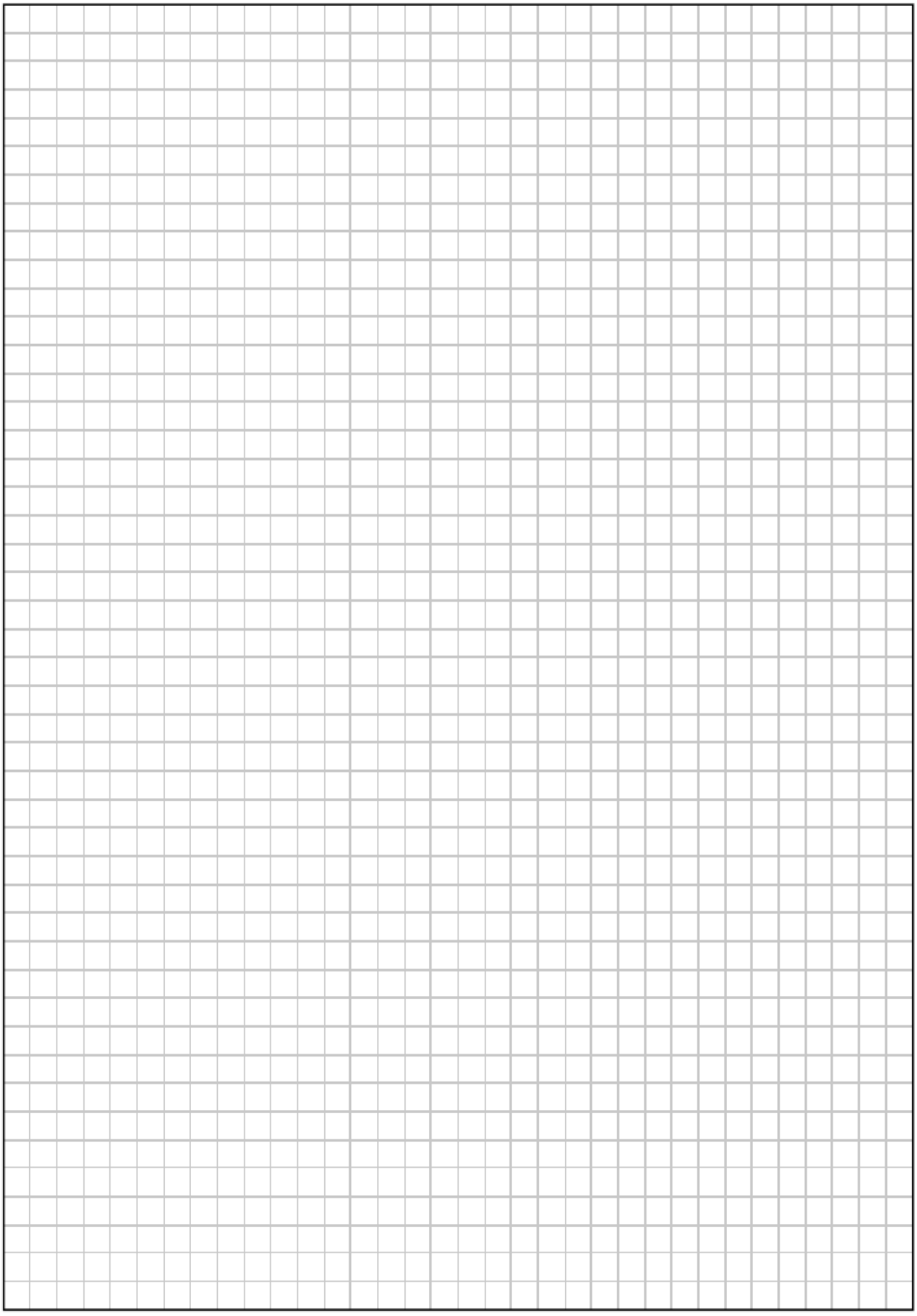
(i) Show that $\sin\beta = \frac{3}{5}$.



(ii) Find the distance $|PR|$.

(b)

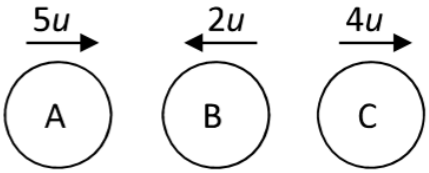
Evaluate $\int e^{2x} \cos 3x \, . \, dx$



Question 6

(a)

Three identical small smooth spheres A, B and C, each of mass m, lie in a straight line on a smooth horizontal surface with B between A and C.



Spheres A and B are projected towards each other with speeds $5u$ and $2u$ respectively, and at the same time C is projected along the line from B away from B with speed $4u$.

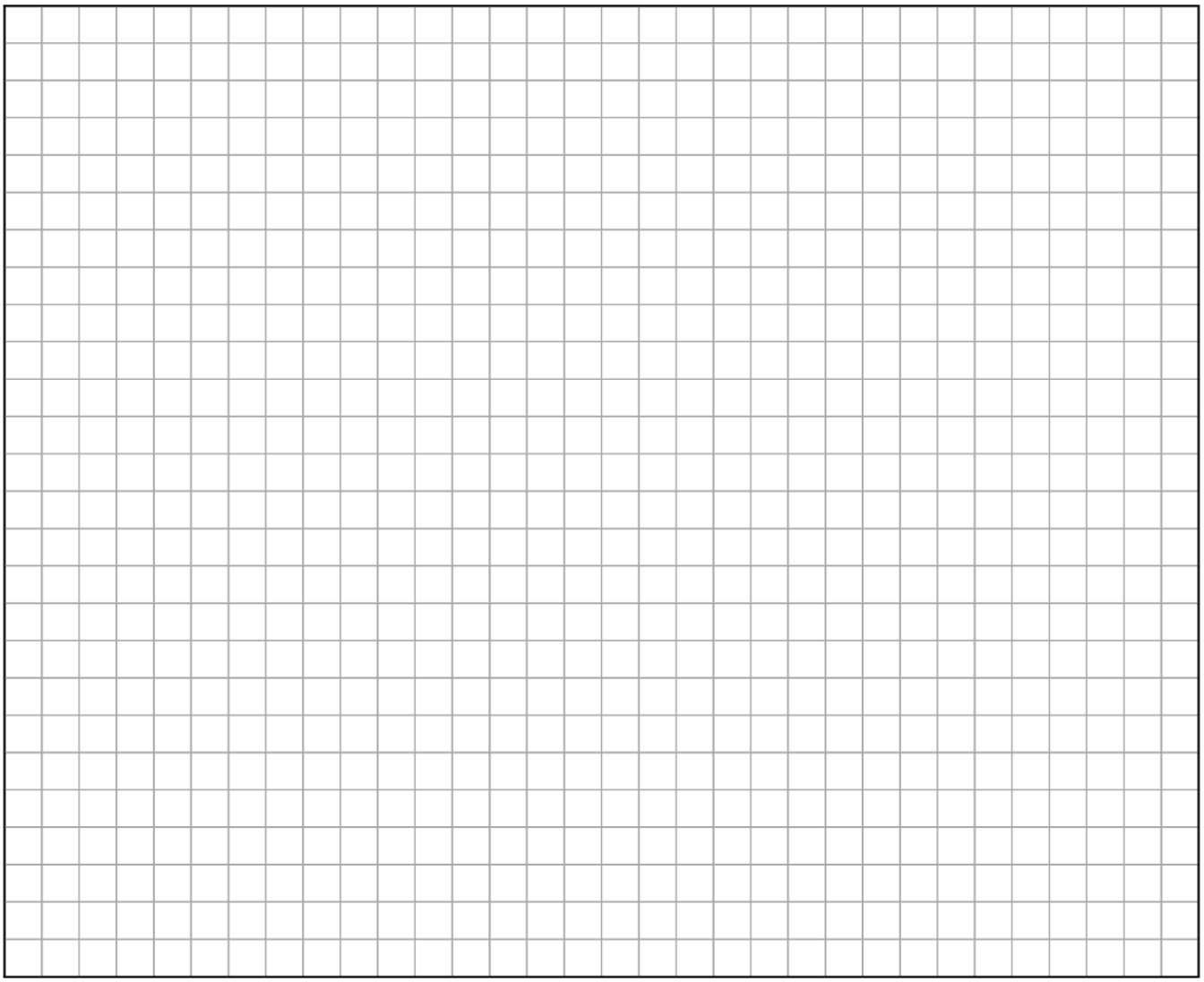
The coefficient of restitution between each pair of spheres is e.

After the collision between A and B there is a collision between B and C.

(i) Find, in terms of e and u, the speed of each sphere after the first collision.

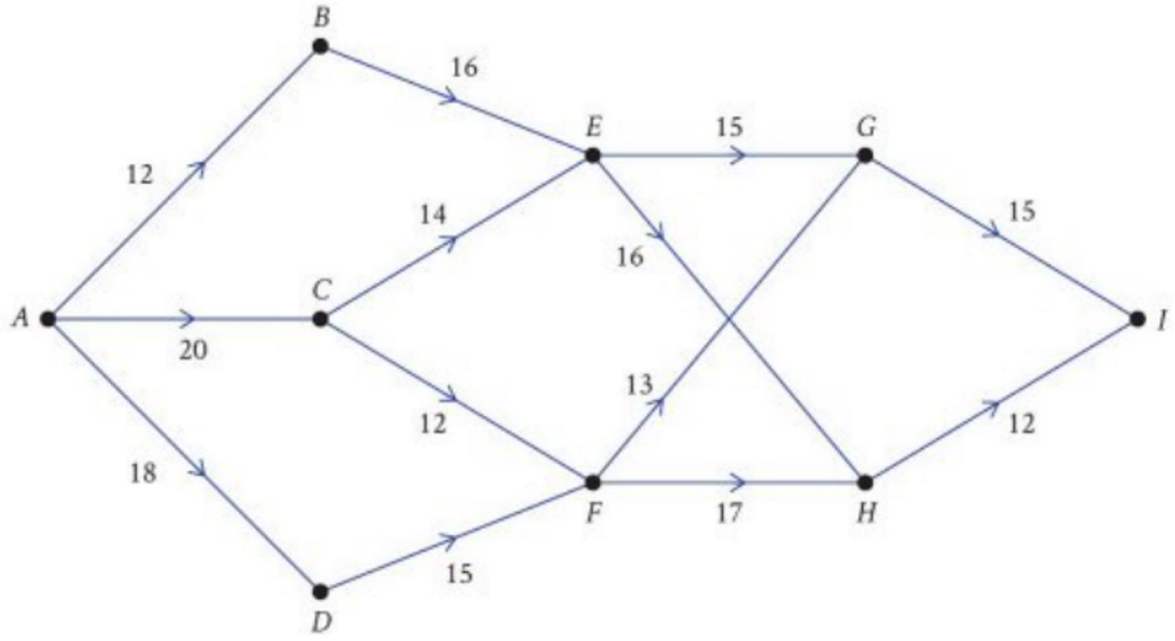
(ii) Show $e > \frac{5}{7}$.

(iii) If $e = \frac{6}{7}$ show that B will not collide with A again.



(b)

The network below shows a system of one way roads. The number on each edge represents the number of bags for recycling that can be collected by driving along that road. A collector is to drive from A to I.



(i) Using the table below, find the maximum number of bags that can be collected driving from A to I.

Stage	State	Action	Destination	Value

(ii) State the route that the collector should take in order to collect the maximum number of bags.

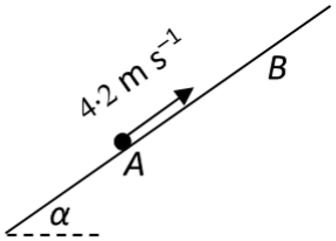
Question 7

(a)

A particle P, of mass 3 kg, is projected along a rough inclined plane from the point *A* with speed 4·2 m s^{-1}. The particle comes to instantaneous rest at *B*.

The plane is inclined at an angle α to the horizontal where $\tan\alpha = \frac{9}{40}$.

The coefficient of friction between the particle and the plane is $\frac{3}{20}$.



(i) Show that the deceleration of P is $\frac{15g}{41}$.

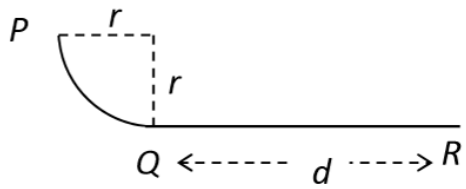
(ii) Find |*AB*|.

After reaching *B* the particle slides back down the plane.

(iii) Find the speed of P as it passes through *A* on its way back down the plane.

(b)

A baggage chute has two sections, *PQ* and *QR*, as shown in the diagram.
PQ is smooth and is a quarter circle of radius *r*.
QR, of length *d*, is rough and horizontal.
The coefficient of friction between the bag and section *QR* is μ.



A bag of mass *m* kg is released from rest at *P* and comes to rest at *R*.

Find

(i) the speed of the bag at *Q* in terms of *r*

(ii) *d* in terms of μ and *r*.

The speed of the bag when it is halfway along QR is 7 m s^{-1}**.**

(iii) Find the value of r.

Question 8

(a)

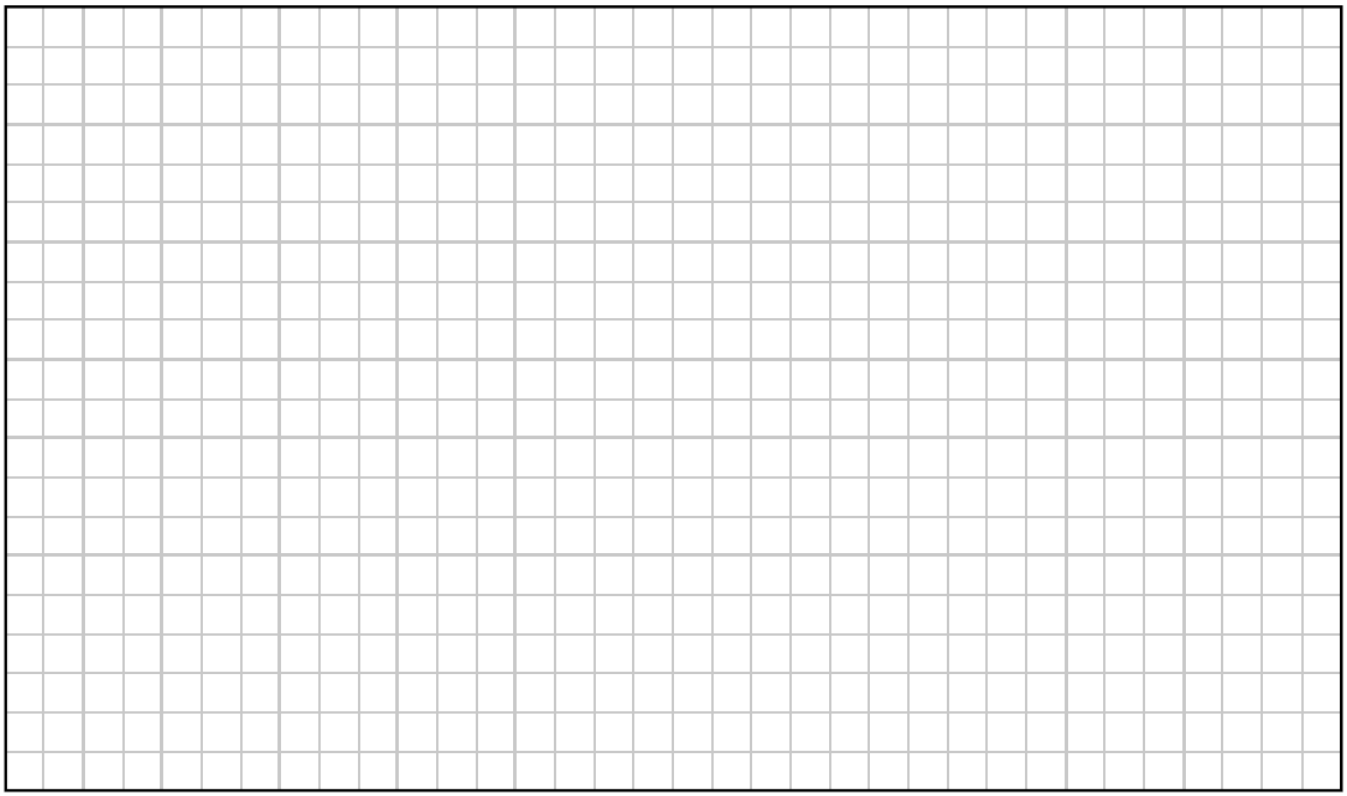
A particle P moves along a straight line.
The speed of P at time t is v, where $v = at^2 + bt + c$ and a, b and c are constants.
The initial speed of the particle is 15 $\mathrm{m\ s^{-1}}$.
After 2·5 seconds the particle reaches its **minimum** speed of 2·5 $\mathrm{m\ s^{-1}}$.

Find

(i) the value of a, the value of b, and the value of c

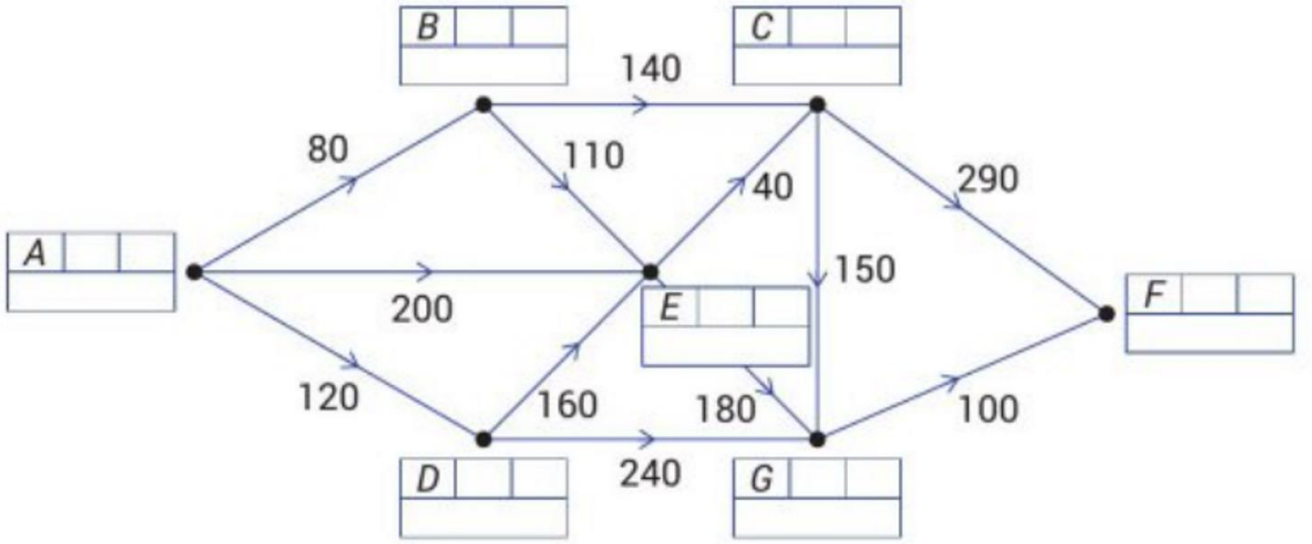
(ii) the acceleration of P when $t = 4$ seconds

(iii) the distance travelled by P in the third second of the motion.



(b)

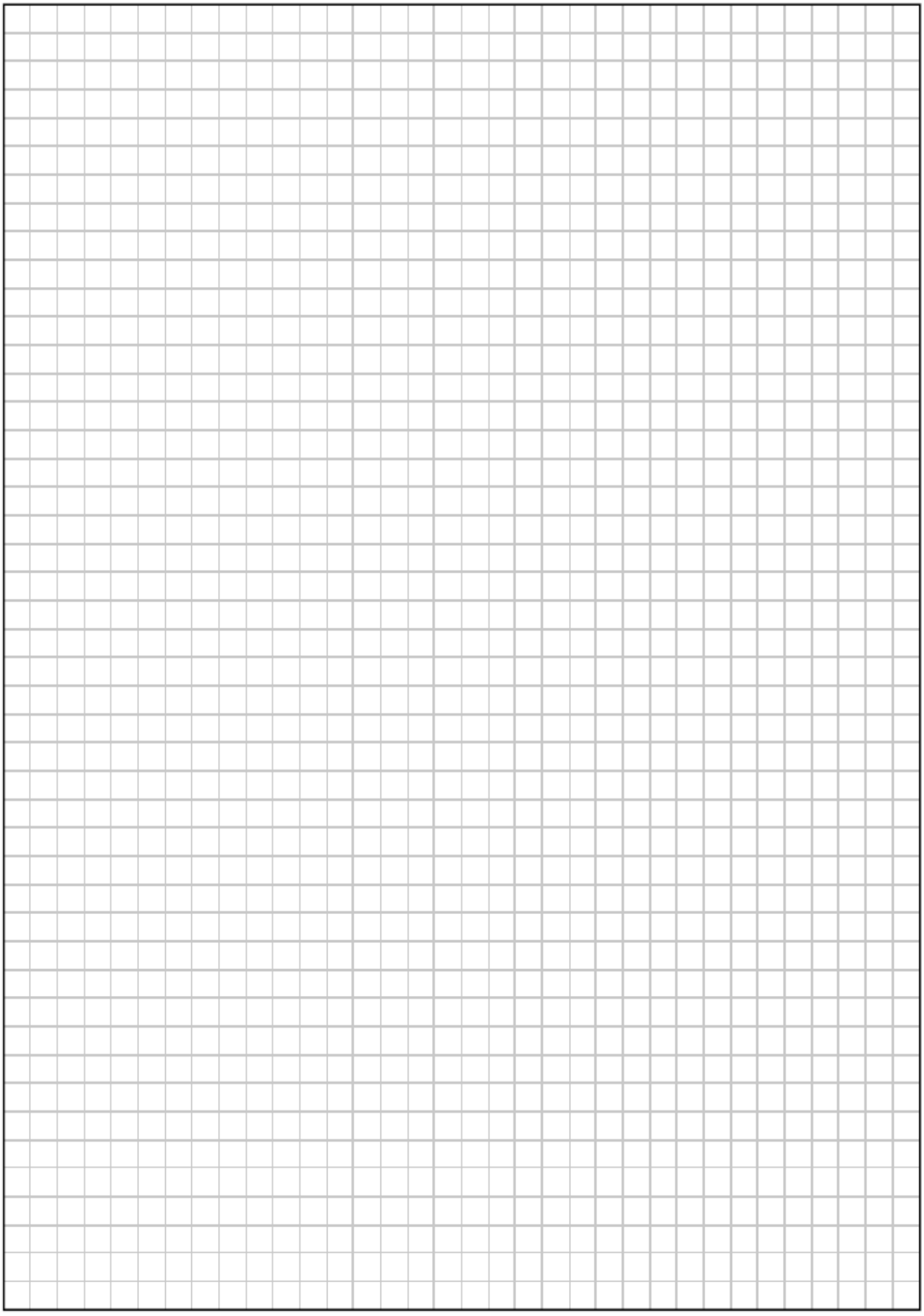
(i) The directed graph below represents a network of one-way streets. The numbers represent the lengths (in metres) of the streets. Find, using an algorithm from your course, the shortest route from A to F and write down its length.



(ii) If the street between D and G is closed for repairs, what is the shortest route from A to F and write down its length.

Page for extra work.

Label any extra work clearly with the question number and part.



Page for extra work.

Label any extra work clearly with the question number and part.