Sample Paper 1 Marking Scheme

Question 1

(a)

A car of mass 1200 kg starts from rest and travels along a straight horizontal road. The engine of the car exerts a constant power of 3000 W.

If there is no resistance to the motion of the car, find

(i) the speed of the car after 3 minutes

(ii) the average speed of the car during this time.

(i) $F = \frac{P}{v} = \frac{3000}{v}$ (5)

$$1200 \times \frac{dv}{dt} = \frac{3000}{v}$$

$\int v\, dv = \frac{5}{2}\int dt$ (5)

$\left[\frac{1}{2}v^2\right]_0^v = \frac{5}{2}[t]_0^{180}$ (5)

$$\frac{1}{2}v^2 = 450$$

$v = 30 \text{ m s}^{-1}$ (5)

(ii) $$1200 \times v\frac{dv}{ds} = \frac{3000}{v}$$

$$\int v^2\, dv = \frac{5}{2}\int ds$$

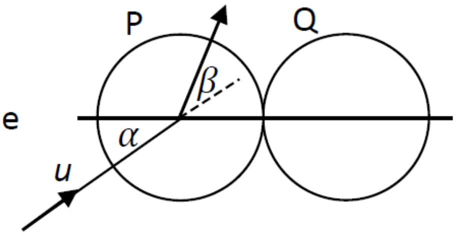
$$\left[\frac{1}{3}v^3\right]_0^{30} = \frac{5}{2}[s]_0^s$$

$$s = 3600$$

$\text{average speed} = \frac{3600}{180} = 20 \text{ m s}^{-1}$ (5) (25)

(b)

A smooth sphere P has mass *m* and speed *u*. It collides obliquely with a smooth sphere Q, of mass *m*, which is at rest. Before the collision, the direction of P makes an angle α with the line of centres, as shown in the diagram.



The coefficient of restitution between the spheres is $\frac{1}{3}$.

During the impact the direction of motion of P is turned through an angle β.

Show that $\tan\beta = \frac{2\tan\alpha}{1+3\tan^2\alpha}$.

P	m	$u\cos\alpha\ \vec{i} + u\sin\alpha\ \vec{j}$	$v_1\ \vec{i} + u\sin\alpha\ \vec{j}$
Q	m	$0\ \vec{i} + 0\ \vec{j}$	$v_2\ \vec{i} + 0\ \vec{j}$

PCM $\quad mu\cos\alpha + m(0) = mv_1 + mv_2$ (5)

NEL $\quad v_1 - v_2 = -\frac{1}{3}u\cos\alpha$ (5)

$$v_1 + v_2 = u\cos\alpha$$

$$v_1 - v_2 = -\frac{1}{3}u\cos\alpha$$

$$v_1 = \frac{1}{3}u\cos\alpha$$ (5)

$$\tan(\alpha+\beta) = \frac{u\sin\alpha}{v_1} = 3\tan\alpha$$ (5)

$$\frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta} = 3\tan\alpha$$

$$\tan\alpha + \tan\beta = 3\tan\alpha - 3\tan^2\alpha\tan\beta$$

$$\tan\beta = \frac{2\tan\alpha}{1+3\tan^2\alpha}$$ (5) (25)

Question 2

(a)

A train takes 40 minutes to travel from rest at station A to rest at station B. The distance between the stations is 20 km. The train left station A at 10:00. At 10:15 the speed of the train was 32 km h^{-1} and at 10:30 the speed was 48 km h^{-1}.

The speed of 48 km h^{-1} was maintained until the brakes were applied, causing a uniform deceleration which brought the train to rest at B.

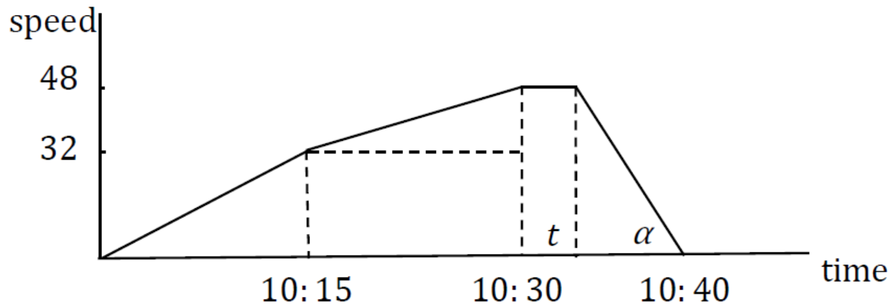
During the first and second 15-minute intervals the accelerations were constant.

(i) Draw a speed-time graph of the motion.

(ii) Find the time taken for the first 16 km.

(iii) Find the deceleration of the train.

(i)



(5)

(ii) $\frac{1}{2}\times\frac{1}{4}\times 32+\frac{1}{4}\times 32+\frac{1}{2}\times\frac{1}{4}\times 16=14$ (5)

$$14+48\left(\frac{t_1}{60}\right)=16$$

$t_1=2.5$ minutes

time = 32.5 minutes (5)

(iii) $14+48\left(\frac{t}{60}\right)+\frac{1}{2}\times\left(\frac{10-t}{60}\right)\times 48=20$

$$\frac{4}{5}t+\frac{2}{5}(10-t)=6$$

$t=5$ min (5)

deceleration = $\tan\alpha=48\div\frac{5}{60}$

$=576\ \text{km h}^{-2}=\frac{2}{45}\ \text{m s}^{-2}$ (5) (25)

(b)

(i) $(1.004)^{12} = 1.049070208$
$\text{APR} \approx 4.907\%$ (5)

(ii) $D_n = 1.004D_{n-1} - A$ (5)

(iii) $D_0 = 120000$
$D_1 = (1.004)(120000) - A$
$D_2 = (1.004)^2(120000) - (1.004)(A) - A$
$D_3 = (1.004)^3(120000) - (1.004)^2(A) - (1.004)(A) - A$ (5)
$D_n = 120000(1.004)^n$
$-[A + (1.004)(A) + (1.004)^2(A) + \cdots + (1.004)^{n-1}(A)]$
$= 120000(1.004)^n - [S_n, a = A, r = 1.004]$
$= 120000(1.004)^n - \frac{(A)(1-(1.004)^n)}{1-1.004}$
$= 250A + (120000 - 250A)(1.004)^n$ (5)

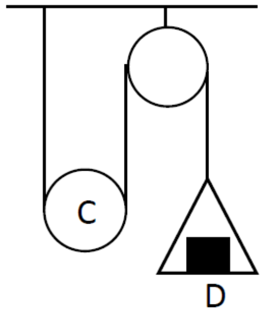
(iv) $0 = 250A + (120000 - 250A)(1.004)^{180}$
$A \approx €936.50$ (5)

Question 3

(a)

The diagram shows a light inextensible string having one end fixed, passing under a smooth movable pulley C of mass *km* kg and then over a fixed smooth pulley. The other end of the string is attached to a light scale pan. A bock D of mass *m* kg is placed symmetrically on the centre of the scale pan.

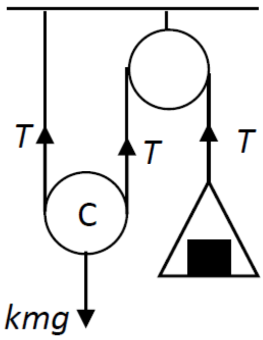
The system is released from rest. The scale pan moves upwards.



(i) Show that $k > 2$.

(ii) Find, in terms of k and m, the tension in the string.

(iii) Find, in terms of k and m, the reaction between D and the scale pan.



(i) $kmg - 2T = kma$ (5)

$T - mg = m \times 2a$ (5)

$a = \left(\frac{k-2}{k+4}\right) g$

$a > 0 \quad \Rightarrow \quad k > 2$ (5)

(ii) $T = mg + 2m\left(\frac{k-2}{k+4}\right) g$

$T = \left(\frac{3k}{k+4}\right) mg$ (5)

(iii) $R - mg = m \times 2a$

$R = \left(\frac{3k}{k+4}\right) mg$ (5) (25)

(b)

(i)

$$\int x^2 \ln x\, dx = \int u\, dv, u = \ln x\,, dv = x^2\, dx \qquad (5)$$

$$du = \frac{1}{x} dx$$

$$v = \frac{1}{3} x^3 \qquad (5)$$

$$\int x^2 \ln x\, dx = (\ln x)\left(\frac{1}{3}x^3\right) - \int \left(\frac{1}{3}x^3\right)\left(\frac{1}{x}\, dx\right) \qquad (5)$$

$$= \frac{x^3}{9}(3\ln x - 1) + c \qquad (5)$$

(ii)

$$F(x) = 20x$$

$$W = \int_0^4 20x\, dx = 10x^2\, |_0^4 = 160 \text{ J} \qquad (5)$$

Question 4

(a)

A particle is projected from a point on horizontal ground. The speed of projection is 14 m s^{-1} at an angle α to the horizontal.

Find the two values of α that will give a range of 10 m.

$$14 \sin\alpha \times t - \frac{1}{2} g t^2 = 0 \qquad (5)$$

$$t = \frac{28 \sin\alpha}{g}$$

$$14 \cos\alpha \times t = 10 \qquad (5)$$

$$14 \cos\alpha \times \frac{28 \sin\alpha}{g} = 10 \qquad (5)$$

$$\sin 2\alpha = \frac{1}{2}$$

$$\alpha = 15° \text{ or } 75° \qquad (5) \qquad (20)$$

(b)

A 60 gram mass is projected vertically upwards with an initial speed of 15 m s^{-1} and half a second later a 40 gram mass is projected vertically upwards from the same point with an initial speed of 22·65 m s^{-1}.

(i) Calculate the height at which the masses will collide.

The masses coalesce on colliding.

(ii) Find the greatest height which the combined mass will reach.

(i)

$$15t - 4.9t^2 = 22.65\left(t - \tfrac{1}{2}\right) - 4.9\left(t - \tfrac{1}{2}\right)^2 \qquad (5)$$

$$12.55t = 12.55$$
$$t = 1 \qquad (5)$$

$$h = 15 \times 1 + \tfrac{1}{2}(-9.8)1^2$$
$$h = 10.1 \text{ m} \qquad (5)$$

(ii)

$$v_1 = 15 - 9.8(1)$$
$$v_1 = 5.2$$

$$v_2 = 22.65 - 9.8\left(1 - \tfrac{1}{2}\right)$$
$$v_2 = 17.75 \qquad (5)$$

$$0.06 \times 5.2 + 0.04 \times 17.75 = 0.1v$$
$$1.022 = 0.1v$$
$$v = \frac{1.022}{0.1} = \frac{511}{50} = 10.22 \qquad (5)$$

$$v^2 = u^2 + 2as$$
$$0 = 10.22^2 + 2(-9.8)s$$

$$s = 5.3$$

$$\text{greatest height} = 5.3 + 10.1 = 15.4 \text{ m} \qquad (5) \qquad (30)$$

Question 5

(a)

Subject	Days Available	Days Allocated	Days Left	% Increase	
App Maths	0	0	0	0*	
	1	1	0	12*	
	2	2	0	20*	
	3	3	0	25*	(5)
Economics	0	0	0	0*	
	1	1	0	9	
		0	1	12*	
	2	2	0	17	
		1	1	21*	
		0	2	20	
	3	3	0	27	
		2	1	29*	
		1	2	29*	
		0	3	25	(5)
Classic St	3	3	0	26	
		2	1	33*	
		1	2	32	
		0	3	29	(5)

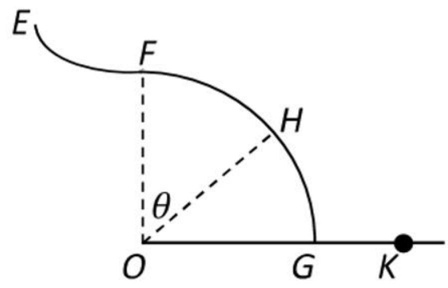
Answer: 2 days Classical Studies, 1 day Applied Maths (33%) (5)

(b)

A smooth slide *EFG* is in the shape of two arcs, *EF* and *FG*, each of radius *r*. The centre *O* of arc *FG* is vertically below *F* as shown in the diagram.

Point *E* is at a height $\frac{r}{5}$ above point *F*.

A child starts from rest at *E*, moves along the slide past the point *F* and loses contact with the slide at point *H*. *OH* makes an angle θ with the vertical.



(i) Find the value of θ.

The child lands in a sandpit at point *K*.

(ii) Find, in terms of *r*, the speed of the child at *K*.

(i) *H* $\frac{1}{2}mv^2 = mg\left\{\frac{1}{5}r + (r - r\cos\theta)\right\}$ (5)

$v^2 = 2gr\left\{\frac{1}{5} + (1 - \cos\theta)\right\}$

$v^2 = 2gr\left\{\frac{6}{5} - \cos\theta\right\}$

H $mg\cos\theta - R = \frac{mv^2}{r}$ (5)

$mg\cos\theta - 0 = 2mg\left\{\frac{6}{5} - \cos\theta\right\}$ (5)

$\cos\theta = \frac{4}{5}$

$\Rightarrow \quad \theta = 36.87°$ (5)

(ii) *K* $\frac{1}{2}m{v_1}^2 = mg\left(r + \frac{1}{5}r\right)$ (5)

${v_1}^2 = \frac{12}{5}gr$

$v_1 = \sqrt{\frac{12}{5}gr}$ **or** $\frac{14\sqrt{3r}}{5}$ **or** $4.85\sqrt{r}$ (5) (30)

Question 6

(a)

A small smooth sphere A, of mass 3*m* moving with speed *u*, collides directly with a small smooth sphere B, of mass *m* moving with speed *u* in the opposite direction. The coefficient of restitution between the spheres is $\frac{1}{2}$.

(i) Find, in terms of *u*, the speed of each sphere after the collision.

After the collision B hits a smooth vertical wall which is perpendicular to the direction of motion of B. The coefficient of restitution between B and the wall is $\frac{2}{5}$.
The first collision between the spheres occurred at a distance 2 metres from the wall. The spheres collide again 4 seconds after the first collision between them.

(ii) Find the value of *u*.

(i) PCM $3m(u) + m(-u) = 3mv_1 + mv_2$ (5)

NEL $v_1 - v_2 = -\frac{1}{2}(u + u)$ (5)

$3v_1 + v_2 = 2u$

$v_1 - v_2 = -u$

$v_1 = \frac{u}{4}$ $v_2 = \frac{5u}{4}$ (5)

(ii) B reaches wall in $2 \div \frac{5u}{4} = \frac{8}{5u}$ seconds

In this time A travels $\frac{u}{4} \times \frac{8}{5u} = \frac{2}{5}$ m

rebound speed of B $\frac{2}{5} \times \frac{5u}{4} = \frac{u}{2}$ (5)

$\frac{u}{4} \times t + \frac{u}{2} \times t = 1.6$

$\frac{3u}{4} \times t = 1.6$

$\frac{3u}{4} \times \left(4 - \frac{8}{5u}\right) = 1.6$

$u = \frac{14}{15} = 0.93$ (5) (25)

(b)

A particle starts from rest and moves in a straight line with acceleration $(25 - 10v)$ m s^{-2}, where v is the speed of the particle.

(i) After time t, find v in terms of t. (Note: $\int \frac{dx}{a+bx} = \frac{1}{b} ln|a + bx| + c$).

(ii) Find the time taken to acquire a speed of 2·25 m s^{-1} and find the distance travelled in this time.

(i)

$$\int \frac{dv}{25\text{-}10v} = \int dt$$ 5

$$\left[-\tfrac{1}{10}\ln(25-10v)\right]_0^V = [t]_0^t$$ 5

$$-\tfrac{1}{10}\ln(25-10v) + \tfrac{1}{10}\ln 25 = t$$

$$\ln\frac{25}{25-10v} = 10t$$

$$v = 2.5\left(1-e^{-10t}\right)$$ 5

(ii)

$$\ln\frac{25}{25-10v} = 10t$$

$$10t = \ln\frac{25}{25-22.5}$$ 5

$$t = \frac{1}{10}\ln 10 = 0.23 \text{ s}$$

$$\int ds = 2.5\int \left(1-e^{-10t}\right)dt$$

$$s = 2.5\left[t + \frac{1}{10}e^{-10t}\right]_0^{0.23}$$

$$= 0.35 \text{ m}$$ 5

(25)

Question 7
(a)

(i) $T = \frac{12m}{6} = 2m$ (5)

$2m = mk(6)$

$k = \frac{1}{3}$ (5)

(ii)

$$\frac{12m}{u} - \frac{mu}{3} = m(7.5)$$ (5)

$$12 - \frac{u^2}{3} = 7.5u$$

$$36 - u^2 = 22.5u$$

$$u^2 + 22.5u - 36 = 0$$

$$2u^2 + 45u - 72 = 0$$

$$(2u - 3)(u + 24) = 0$$

$2u - 3 = 0$ or $u + 24 = 0$

$2u = 3$ or $u = -24$ ✗

$u = \frac{3}{2}$ (5)

(b)

A particle P travelling in a straight line has a deceleration of $4v^{n+1}$ m s^{-2}, where *n* (> 0) is a constant and *v* is its speed at time *t* (> 0).

P has an initial speed of *u*.

(i) Find an expression for *v* in terms of *u*, *n* and *t*.

(ii) When *n* = 3 obtain an expression for the speed of P when it has travelled a distance of 3 m from its initial position.

(i)

$$\frac{dv}{dt} = -4v^{n+1}$$

$$-\int \frac{dv}{v^{n+1}} = 4\int dt \qquad (5)$$

$$\frac{1}{n} \times \left[\frac{1}{v^n}\right]_u^v = 4[t]_0^t \qquad (5)$$

$$\frac{1}{v^n} - \frac{1}{u^n} = 4nt$$

$$v^n = \frac{1}{4nt+\frac{1}{u^n}}$$

$$v = \left(\frac{1}{4nt+\frac{1}{u^n}}\right)^{\frac{1}{n}} = \left(\frac{u^n}{4ntu^n+1}\right)^{\frac{1}{n}} = \frac{u}{(4ntu^n+1)^{\frac{1}{n}}} \qquad (5)$$

(ii)

$$v\frac{dv}{ds} = -4v^4$$

$$-\int \frac{dv}{v^3} = 4\int ds \qquad (5)$$

$$\left[\frac{1}{2v^2}\right]_u^v = [4s]_0^3 \qquad (5)$$

$$\frac{1}{2v^2} - \frac{1}{2u^2} = 12 - 0$$

$$\frac{1}{2v^2} = \frac{1}{2u^2} + 12 = \frac{1+24u^2}{2u^2}$$

$$v^2 = \frac{u^2}{1+24u^2}$$

$$v = \frac{u}{\sqrt{1+24u^2}} \qquad (5) \qquad (30)$$

Question 8

(a)

A small smooth sphere A, of mass 1·5 kg, moving with speed 6 m s^{-1}, collides directly with a small smooth sphere B, of mass m kg, which is at rest.
After the collision the spheres move in opposite directions with speeds v and $2v$, respectively.
80% of the kinetic energy lost by A as a result of the collision is transferred to B.
The coefficient of restitution between the spheres is e.

Find **(i)** the value of v

(ii) the value of e.

(i) PCM $1.5(6) + m(0) = 1.5(-v) + m(2v)$ — 5

$$m = \frac{9 + 1.5v}{2v}$$

$$0.8\left\{\tfrac{1}{2}(1.5)(6^2) - \tfrac{1}{2}(1.5)v^2\right\} = \tfrac{1}{2}m(2v)^2$$ — 5

$$21.6 - 0.6v^2 = 9v + 1.5v^2$$

$$2.1v^2 + 9v - 21.6 = 0$$

$$v = \frac{12}{7} \text{ m s}^{-1}$$ — 5

(ii) NEL $v_1 - v_2 = -e(6 - 0)$ — 5

$$-v - 2v = -6e$$

$$e = \frac{v}{2} = \frac{6}{7}$$ — 5

25

(b)

$v_0 = 2g$ m/s

$F = ma$

$-mg - mkv^2 = ma$

$a = -g - kv^2$ (5)

$v\frac{\mathrm{d}v}{\mathrm{d}x} = -g - kv^2$

$\int_0^h 1\,\mathrm{d}x = -\int_{2g}^0 \frac{v}{g+kv^2}\,\mathrm{d}v = \int_0^{2g} \frac{v}{g+kv^2}\,\mathrm{d}v$ (5)

RHS, by substitution:

Let $f(v) = g + kv^2$, then $\mathrm{d}f = 2kv\,\mathrm{d}v.$ (5)

$$\int \frac{v}{g+kv^2}\mathrm{d}v = \int \frac{1}{2kf}\,\mathrm{d}f$$

$$= \frac{1}{2k}\ln f$$

$$= \frac{1}{2k}\ln(g + kv^2)$$ (5)

$$x\Big|_0^h = \frac{1}{2k}\ln(g + kv^2)\Big|_0^{2g}$$

$$h - 0 = \frac{1}{2k}\ln(g + 4kg^2) - \frac{1}{2k}\ln(g + 0)$$

$$h = \frac{1}{2k}\ln(1 + 4kg)$$ (5)

Q.E.D.