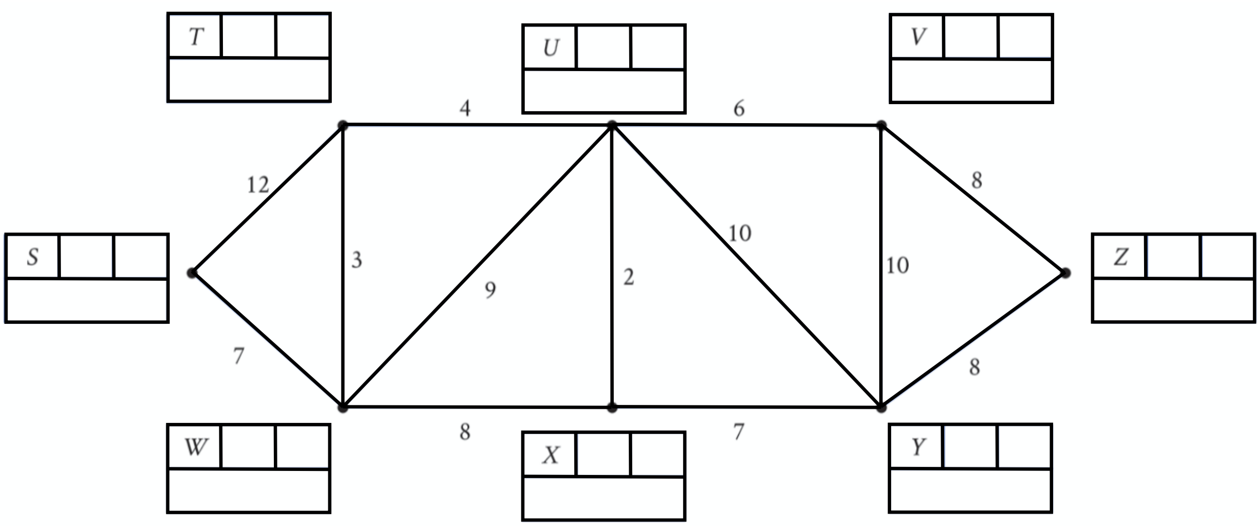
- Chapter 12: Optimal Paths
- Topic 50: Dijkstra's Algorithm
  - If we want to find the shortest path from one node to another, we can use Dijkstra's Algorithm.
  - By "shortest", we could mean shortest distance, quickest time or lowest cost, depending on the weights given in the graph.
  - Used in GPS devices to calculate shortest distances.
  - When presented with a graph, we begin by adding the following to each node:

Node	Order of Labelling	Final Value
Working Values		

- Example: Pg 241 Ex 12A Q2

(i) Use Dijkstra's Algorithm to find the shortest distance from S to Z, given the distances in the network below.

(ii) Name the shortest path.



Solution:

(i)

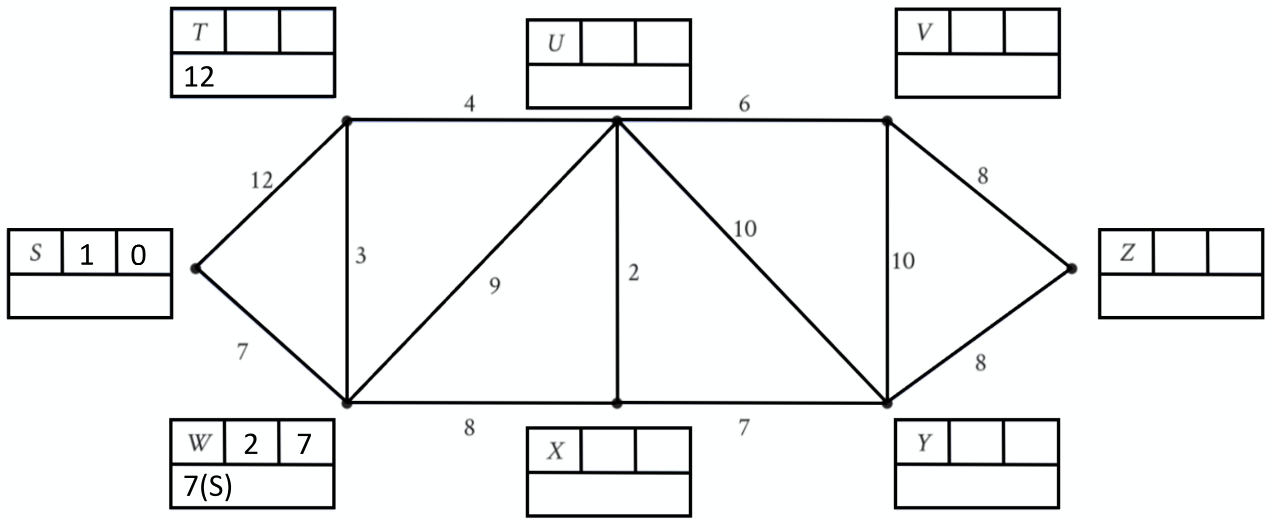
Step 1a: S is our starting point, so we label that first, and add a "1" to the order of labelling box at S. Its distance from S is 0, so we write "0" in the final value of S.

Step 1b: We now fill out the working values for the nodes adjacent to S - in this case, 12 at T and 7 at W.

Step 2a:

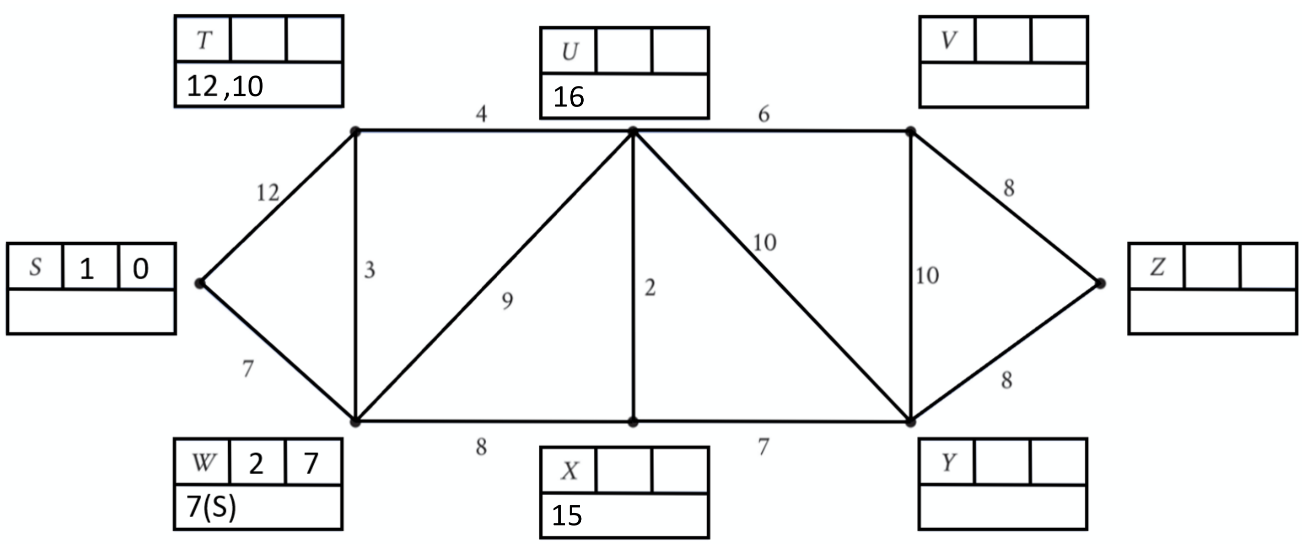
- For our next Final Value, we choose the node that has the SMALLEST working value, which in this case is W.

- So, W becomes the second node to be completed, with a final value of 7…..we write down the node it came from in brackets, as shown.



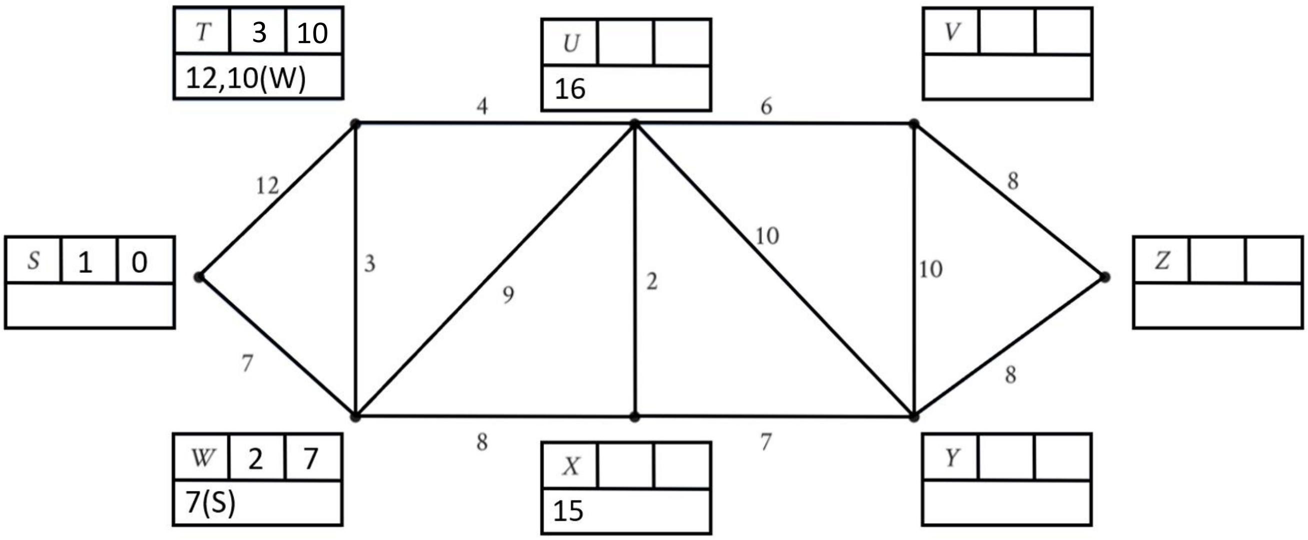
Step 2b: We now fill out the working values for any node which W leads to in one step, which in this case are T, U and X.

- The current working value of T is 12, but we can reach T from W in 7 + 3 = 10, so 10 becomes the new working value for T, as it's smaller than 12.
- W goes to U in a step of 9, so we add 9 to the working value of W, which makes 16 the first working value of U.
- W goes to X in a step of 8, so 15 becomes the first working value for X.



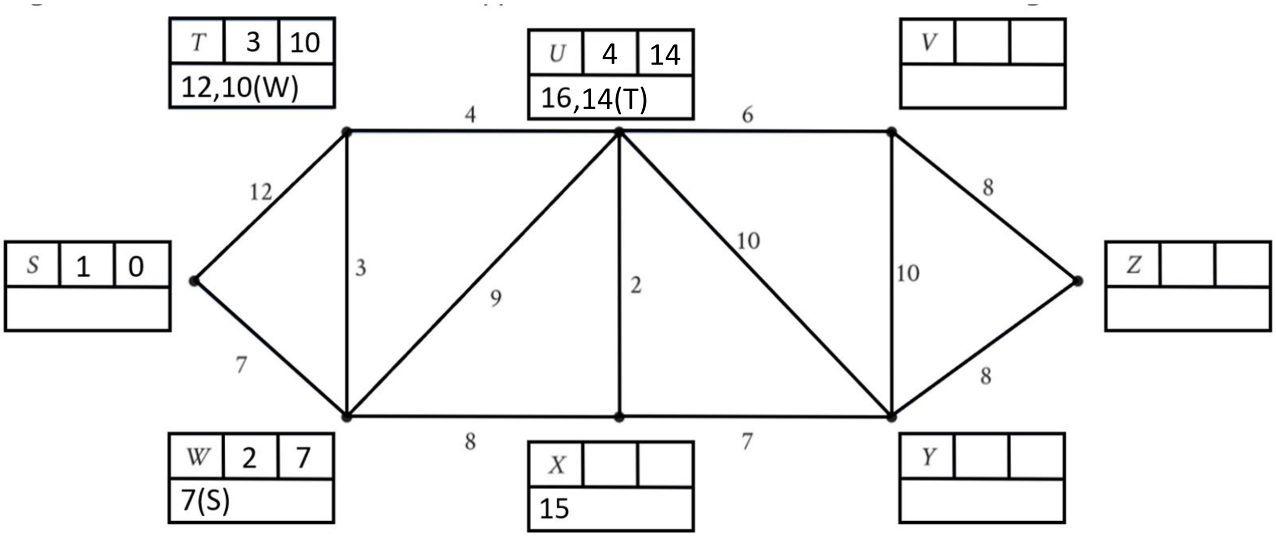
Step 3a: We then look at the uncompleted working values and choose the SMALLEST one again.

- In this case, the lowest working value is T, as 10 is smaller than both 15 at X and the 16 value on U.
- So T becomes or third completed node, with a final value of 10 and we write down again, where it came from.



Step 3b: We now update the working values of any node that is connected to T in one step.

- The only node connected to T is U, which will now have a working value of 10 + 4 = 14.

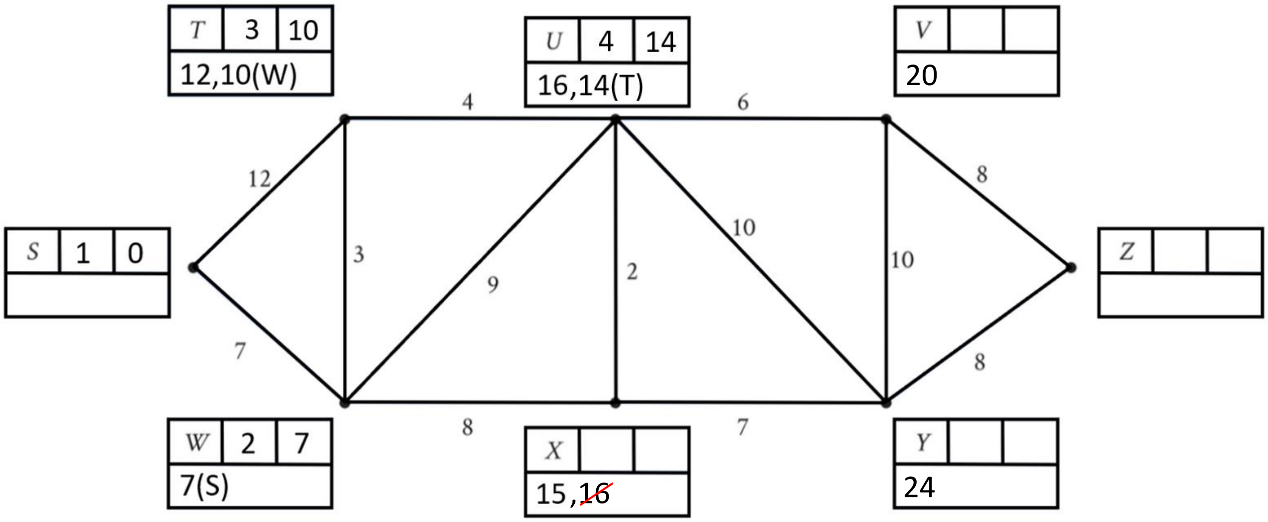


Step 4a: Again, we look now at the uncompleted working values and choose the SMALLEST one again.

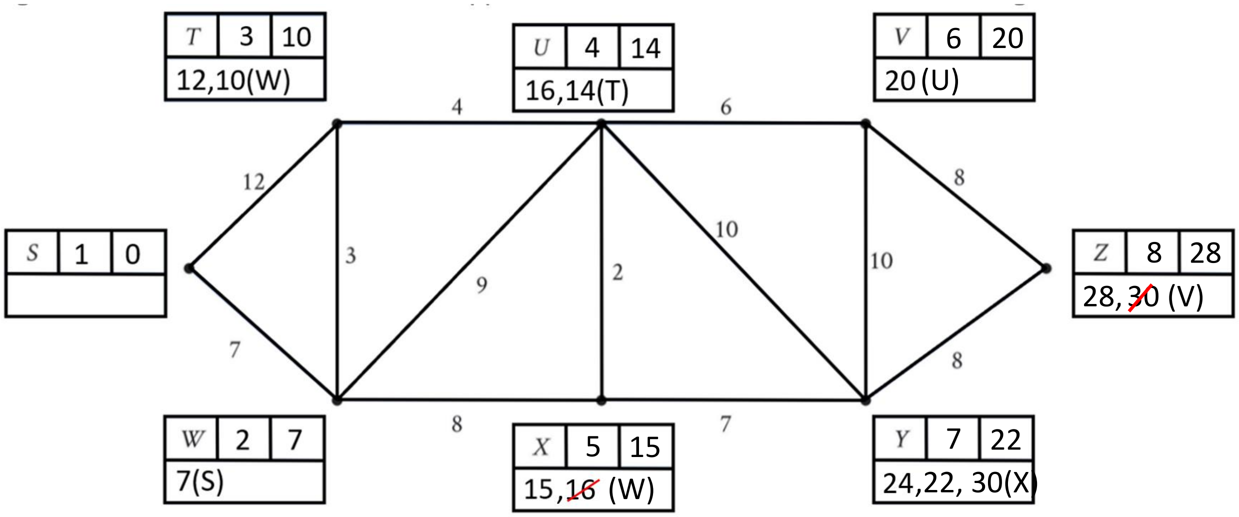
- In this case, the lowest working value is U, so that becomes our 4th completed node with a final value of 14 and we write down where it came from.

Step 4b: Updating the working values of any node that is connected to U in one step, makes V have a working value of 20, X a value of 16 and Y a value of 24.

- As the new working value of 16 for X is bigger than the one that was there already, we can cross out the 16.
- We then continue in the same manner as before.



Step 5: We then continue in this manner to complete as shown below:



(ii) We now work back from Z, to find the shortest path, looking at the nodes they came from, so the shortest path is SWTUVZ, which has a weight of 28.

Note:

- If you were asked for shortest path from S to X, it would be SWX, of length 15.

Classwork Questions: Pg 241/242 Ex 12A Qs 1/4/5/11 (Extra challenge: Q13)

Topic 51: Critical Path Analysis

- When managing large projects, it can be important to plan out the order that activities in the project are done, as some of them have to be finished before others can start.
- Critical Path Analysis can be used in the planning process.
- We begin by making out a precedence table, which lists all the activities that have to be done, and which ones are dependent on previous ones.
- The process starts the source node and ends at the sink node.
- We normally have to use trial and error to get the right activity network, that shows the order activities have to happen.

- Example 1: Pg 248 Ex 12B Q2

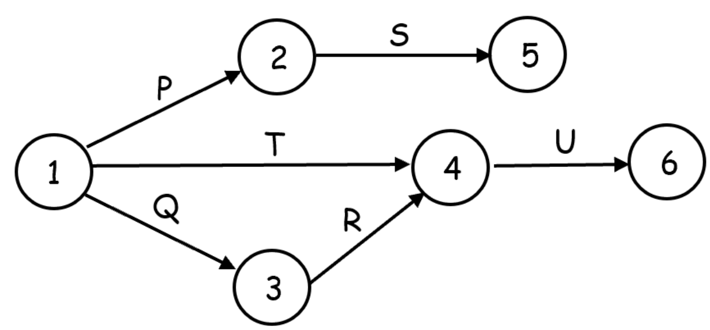
A project consists of 6 activities : P, Q, R, S, T and U. P and Q can start now, but R cannot start until Q is completed. S cannot start until P is completed. U cannot start until both T and R are completed.

(i) Copy and complete the precedence table.

(ii) Draw an activity network for the project.

Solution:

Activity	Depends on…
P	-
Q	-
R	Q
S	P
T	-
U	R, T



Classwork Questions: Pg 248/249 Ex 12B Qs 1/3/5/7/10

Note: Sometimes, some dummy variables are required if an activity requires a previous activity to be completed, without adding a double edge to our activity network.

- If a node in the precedence table doesn't appear later in the table, then it can be the sink node.

- Example 2: Pg 254 Ex 12C Q5

  The precedence table below shows the activities involved in an engineering project.

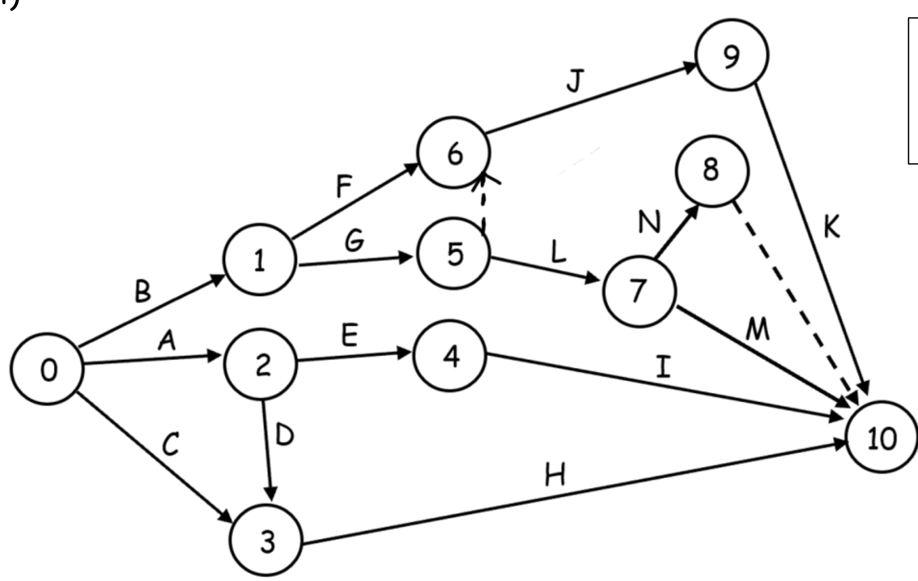
  (i) Draw an activity network for the project, using two dummies.

  (ii) Explain why two dummies are needed.

Activity	Depends on……	Activity	Depends on……
A	-	H	C, D
B	-	I	E
C	-	J	F, G
D	A	K	J
E	A	L	G
F	B	M	L
G	B	N	L

Solution:

(i)



Draw Nodes first and then number them all when finished then.

(ii) Two dummies are shown with dashed lines in the diagram.

- L depends on G and J depends on F and G.
- Both M and N depend on L and must finish in the sink node.

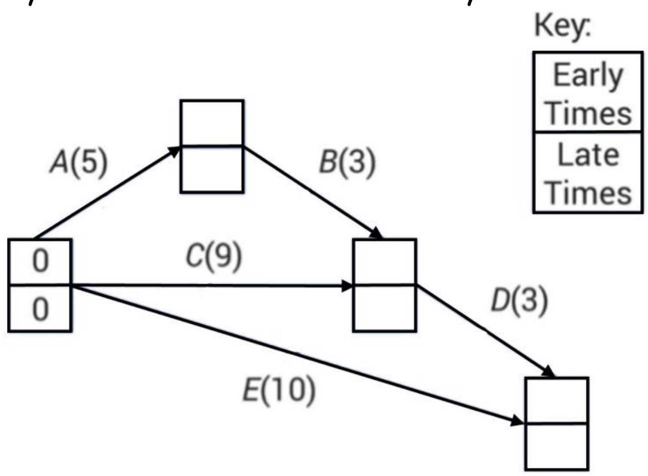
Classwork Questions: Pg 254/255 Ex 12C Qs 1/3/4/7/8

➢ Topic 52: Critical Path/Activities

- The early time of an event is the earliest time that all preceding events are finished.
- The late time of an event is the latest time by which all preceding events have to be completed to prevent delaying the project.
- When presented with an activity network, we begin by working out the early times, by working our way forward through the network (a forward pass/scan).
- We then work our way back from the sink node and work out the late times (a backward pass/scan).

• Example 1: Pg 258 Ex 12D Q5

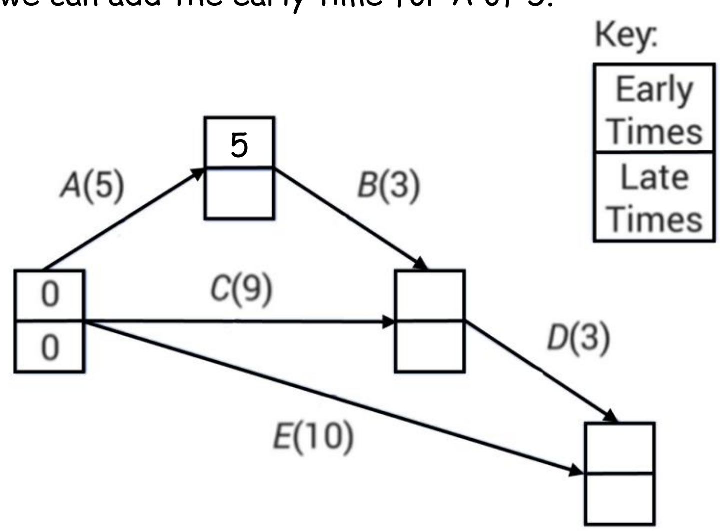
(i) Fill out the early and late times in the activity network below:



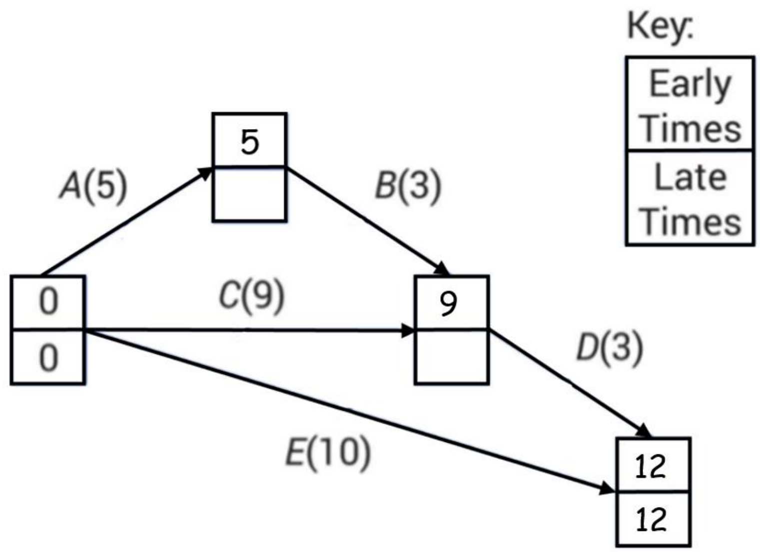
(ii) What is the minimum time for the completion of the whole project (in the time units given)?

Solution:

(i) In the first step, we can add the early time for A of 5.

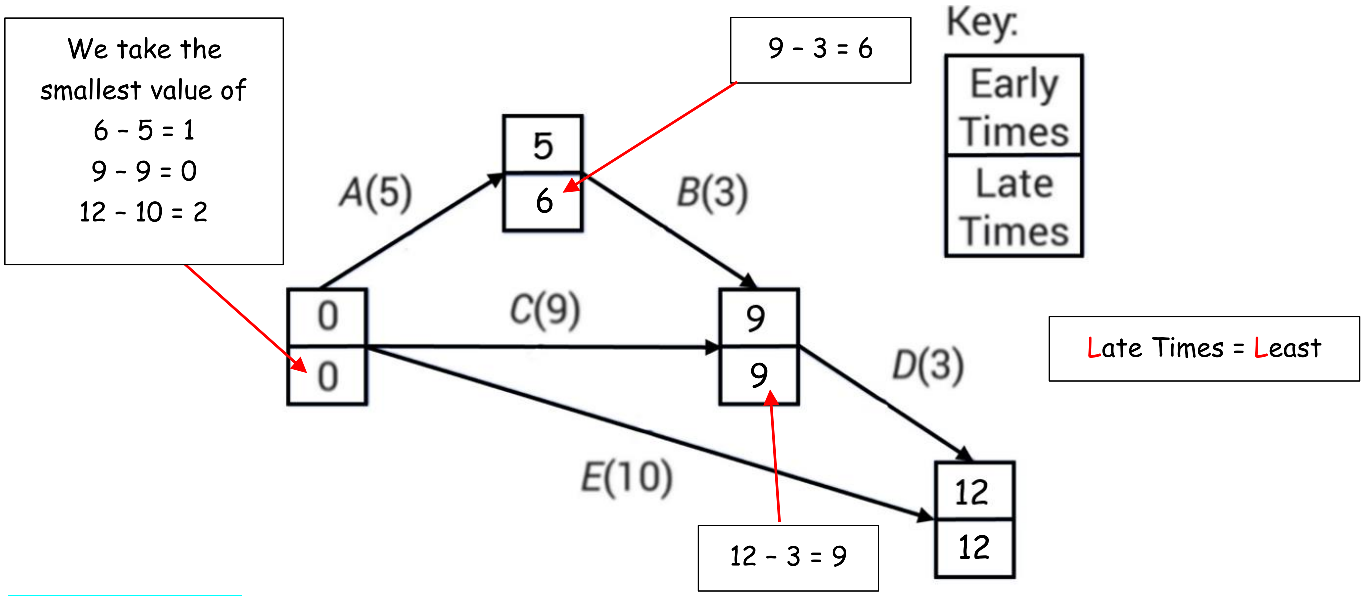


- Looking at the node where B and C meet would give early times of 8 (3 + 5) or 9.
- We take the highest time, as this is the earliest time that BOTH preceding activities could be finished. (Early Times = Enormousest)
- Repeating the same procedure at the node where D and E meet, we add in the biggest early time of 12 (3 + 9), rather than the 10 from activity E.
- We add 12 as the late time on the sink node also.



(ii) The project will take 12 days to complete.

- We now work back from the sink node and add in the late times:

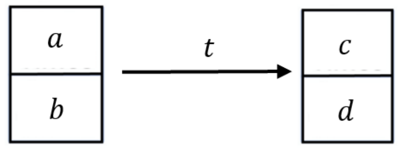


Things to Note:

- Source Node will always be 0, 0 and Sink Node will always be t, t
- In the question above, A could be completed 5 days from the start, but if it were to be delayed to day 6, the project would still be completed in 12 days.

Note: The total float of an activity is the time the start of a project could be delayed, without delaying the entire project.

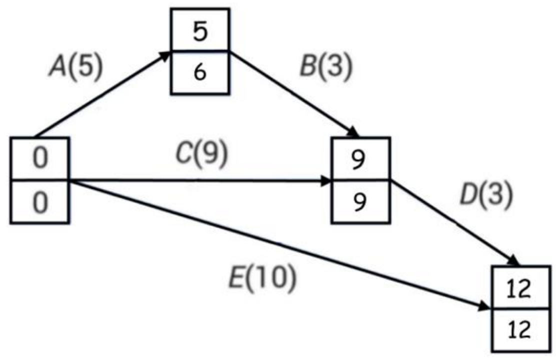
- To calculate the total float for two nodes with early and late times, as shown below:



Can explain with specific figures.

Total Float = $d - a - t$

- Example 1: Find the total float of the activity network on the previous page.



Solution:

Activity	Calculation	Total Float
A	$6 - 0 - 5 = 1$	1
B	$9 - 5 - 3 = 1$	1
C	$9 - 0 - 9 = 0$	0
D	$12 - 9 - 3 = 0$	0
E	$12 - 0 - 10 = 2$	2

- A critical activity has a total float of 0. (Any delay to this activity will hold up the project)
- A critical path is a path from source node to sink node which connects only the critical activities together.
- The critical path in the example above would be CD.

Classwork Questions: Pg 258 Ex 12D Qs 1/2/3/7/9

Classwork Questions: Pg 260/261 Ex 12E Qs 3/5/7

- Topic 53: Gantt Charts
  - A Gantt Chart is a way of displaying an activity network, showing the time taken to complete activities and the total float times of each activity.

- Example: Pg 264 Ex 12F Q2

  This is the activity network for a project, in which the time is measured in hours. The project starts at midnight.

  (a) Identify the critical path in this activity network.

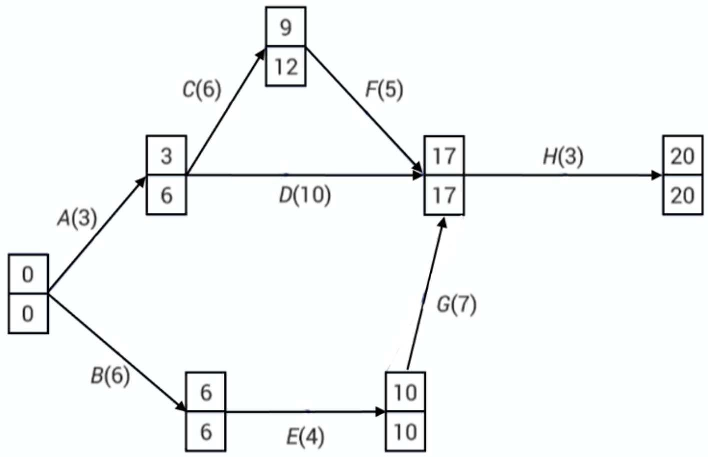
  (b) Draw a cascade chart (Gantt chart).

  (c) What is the total float of (i) D (ii) E and (iii) F?

  (d) An inspector is going to visit at 11:30 hours.

  (i) Which activities **must** be happening at this time?

  (ii) Which activities **may** be happening at this time?



- Solution:

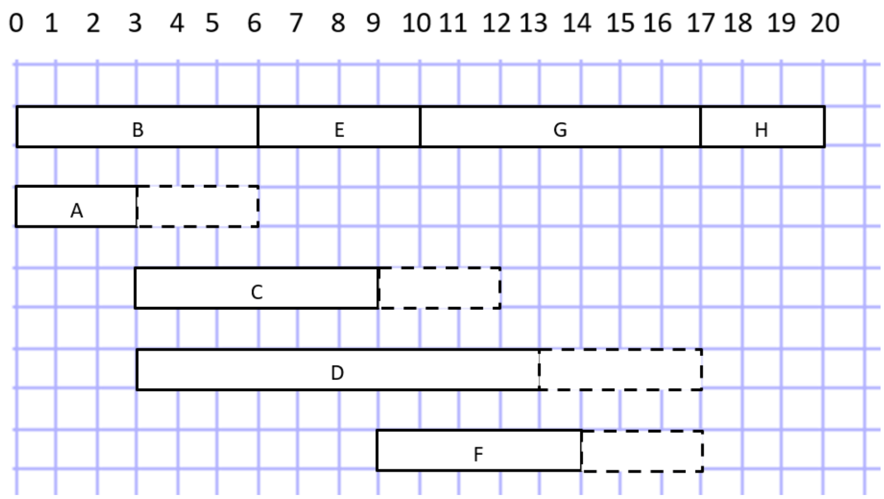
  (a) As we did before, we have to work out the total float of each activity, to find the critical path:

Activity	Calculation	Total Float
A	$6-0-3=3$	3 hrs
B	$6-0-6=0$	0 hrs
C	$12-3-6=3$	3 hrs
D	$17-3-10=4$	4 hrs
E	$10-6-4=0$	0 hrs
F	$17-9-5=3$	3 hrs
G	$17-10-7=0$	0 hrs
H	$20-17-3=0$	0 hrs

  - So, the critical activities are B, E, G and H.
  - This means the critical path is BEGH.

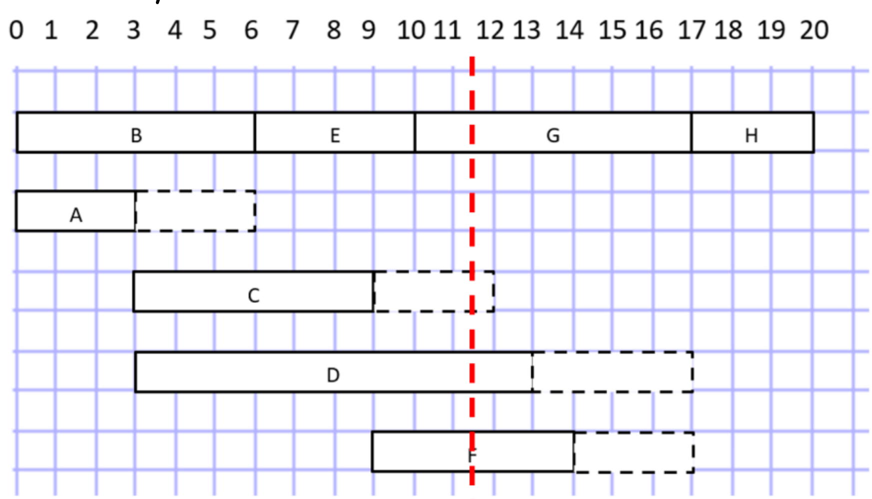
(b) The time taken to complete this project is 20 days, so the Gantt Chart will be 20 units wide:

- It is important that we put any critical paths in first at the top of the chart.
- Then starting with Activity A, we can see its early time is 3, but its late time is 6, so it could end as late as 6 days and not delay the project.
- We show the extra 3 days, or the total float for that activity, with a dashed box.
- Activity C can start at 3 and lasts 6 days, but it can extend on to 12 days if needed, as its total float is 3 days.



(c) The total float times for D, E and F are 4 hrs, 0 hrs and 3 hrs respectively.

(d) We now mark in the key time of 11:30 on our chart



- We can see from the chart that G, D **must** be happening.
- We can also see that C **might** be happening, if it has to run into its additional allowed time.
- F looks like it must be happening, but it could slide to the right and be entirely to the right of the dashed line, so in fact, it **might** be happening also.

=> Must be happening = D, G Might be happening = C, F

Classwork Questions: Pg 264/265 Ex 12F Qs 1/3/6

Topic 54: Scheduling

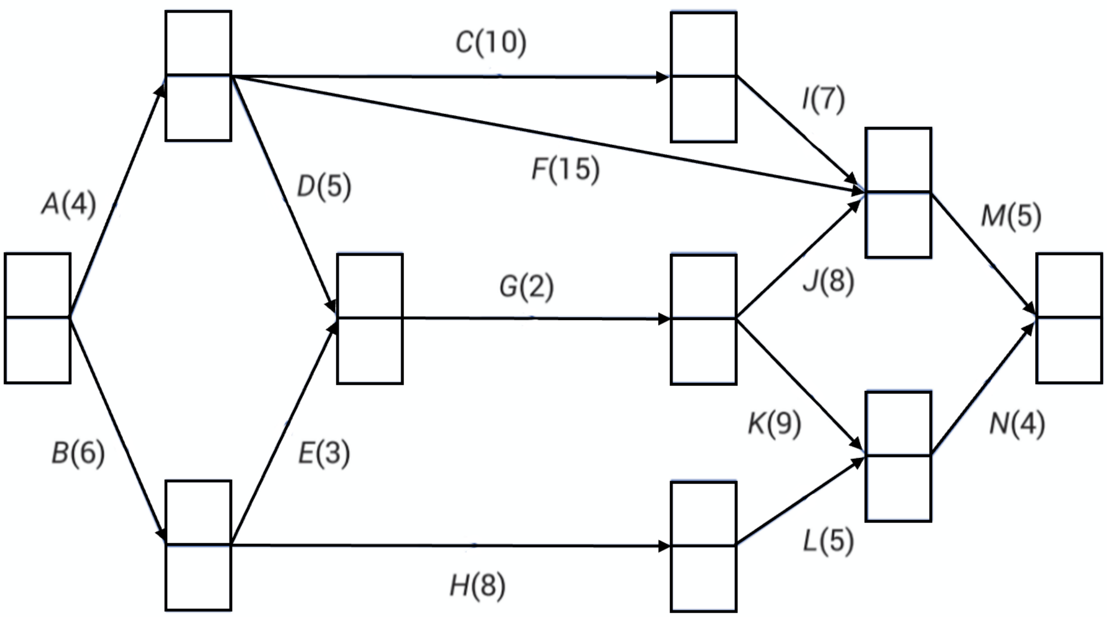
- The process of assigning workers to the various activities in a project, is known as scheduling.
- See Handout for Scheduling
- There are two types of scheduling problems we will look at here:
  a) Minimum Time
  b) Having less than the required number of workers for minimum time

a) Minimum Time:

- Example: Pg 269 Ex 12G Q3

The network in the diagram shows the activities involved in the process. The numbers in brackets represent the time, in days, taken to complete that activity.

(i) Copy this diagram. Calculate the early and late time for each event and fill them into your diagram.



(ii) Determine the critical activities and the length of the critical path.

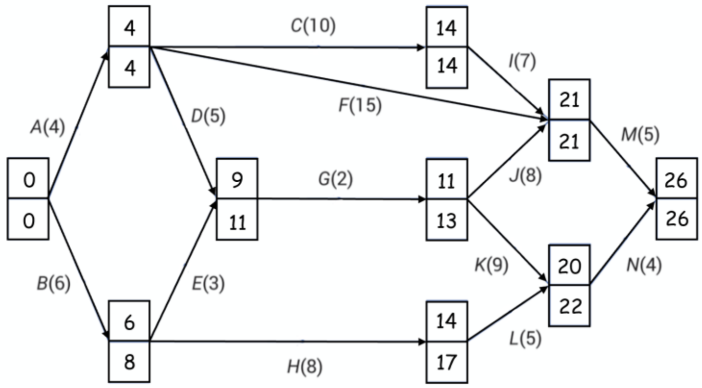
(iii) Draw a Gantt Chart for the process. Each activity requires only one worker, and workers may not share an activity.

(iv) Use your Gantt Chart to determine the minimum number of workers required to complete the process in the minimum time.

(v) Schedule the activities, using the number of workers you found in (iv) so that the process is completed in the shortest time.

Solution:

(i) The early and late times added in would yield the chart below:

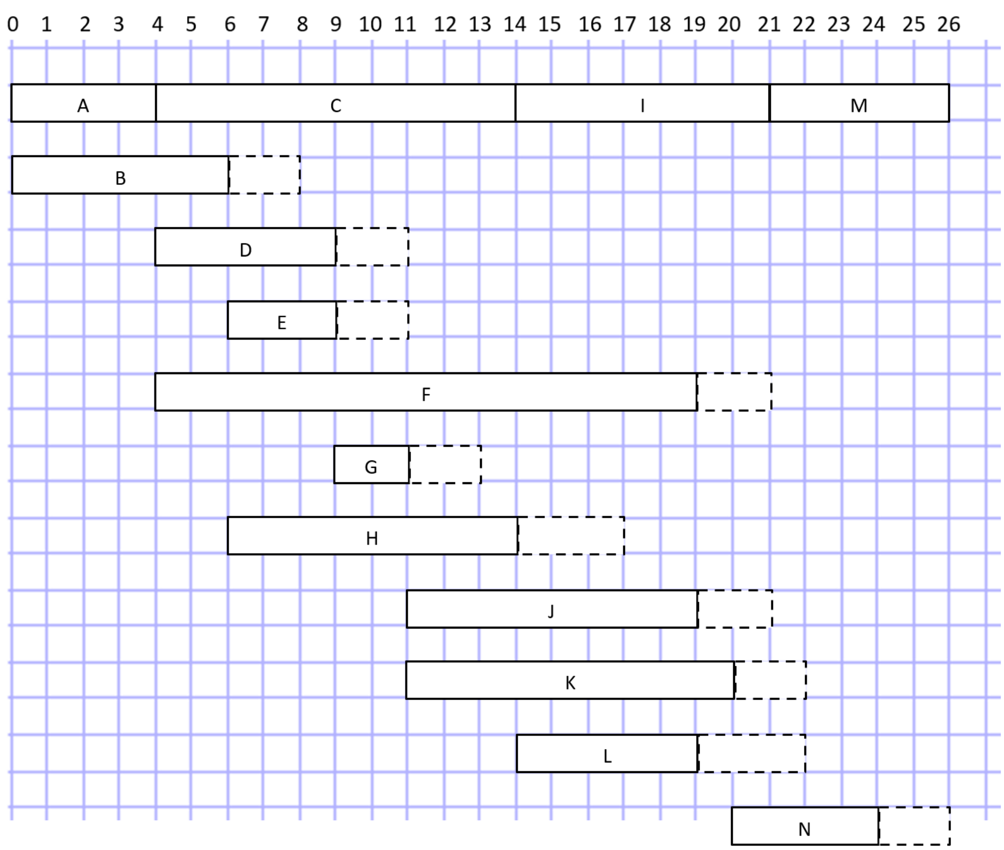


(ii) To find the critical activities and path, we use the total float values again:

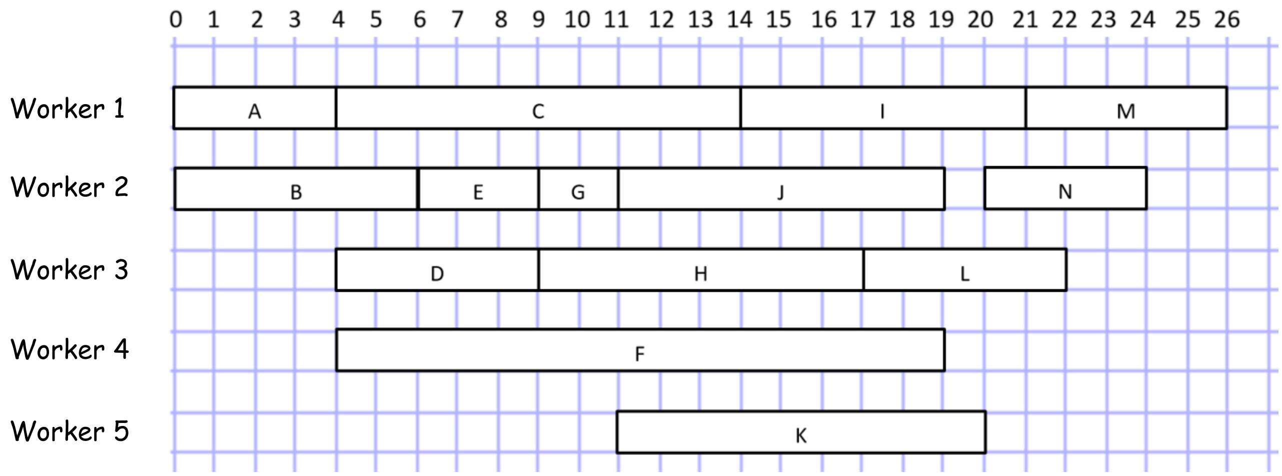
Activity	Calculation	Total Float	Activity	Calculation	Total Float
A	$4-0-4=0$	0 days	H	$17-6-8=3$	3 days
B	$8-0-6=2$	2 days	I	$21-14-7=0$	0 days
C	$14-4-10=0$	0 days	J	$21-11-8=2$	2 days
D	$11-4-5=2$	2 days	K	$22-11-9=2$	2 days
E	$11-6-3=2$	2 days	L	$22-14-5=3$	3 days
F	$21-4-15=2$	2 days	M	$26-21-5=0$	0 days
G	$13-9-2=2$	2 days	N	$26-20-4=2$	2 days

- So, the critical activities are A, C, I, M and the critical path is ACIM, which will take 26 days.

(iii) The Gantt Chart is shown below:



(iv) and (v) One possible scheduling solution is shown below, so we will need 5 workers to complete the project in minimum time.



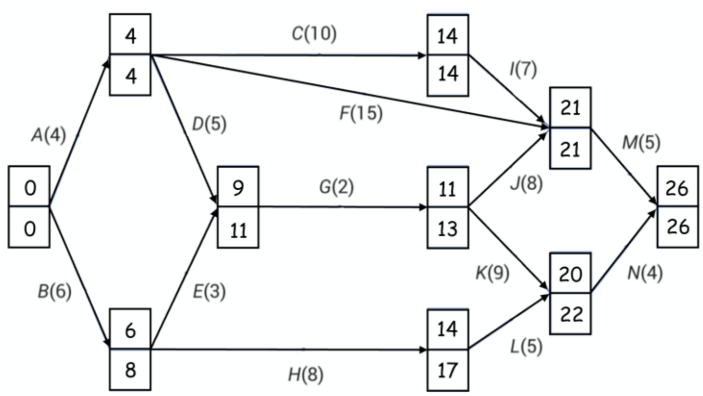
b) <u>Having less than the required number of workers for minimum time:</u>

- Continuing on from yesterday's example, we will now look at the second type of scheduling problem, where we don't have the minimum number of workers.
- This can mean that our project is going to take longer than the minimum time.
- Before we start, we can calculate the lower bound, using the formula below:

$$\text{Lower Bound} = \frac{\text{sum of all activity times}}{\text{critical time for the project}}$$

$$= \frac{4+6+10+\cdots+5+4}{26} = \frac{91}{26} = 3.5$$

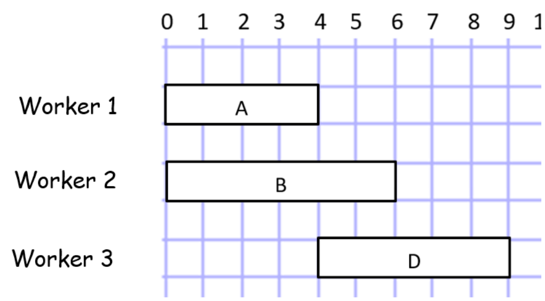
- We always **round this figure up** and it tells us the minimum number of workers that will more than likely be required to complete the job in the minimum time….in this case it would be 4 workers.
- We may need more than this though, as we saw yesterday…..we actually needed 5.
- Now, we're going to schedule the same project assuming we only have 3 workers available.
- The activity network was as shown below:



- The Gantt Chart is of no use to us in this scenario, and we just schedule activities to maximise the use of the 3 workers we have available, trying to minimise any worker being idle as much as possible.
- When assigning activities, we assign them in order of the **lowest late times and in alphabetical order, if two or more activities have the same late times.**

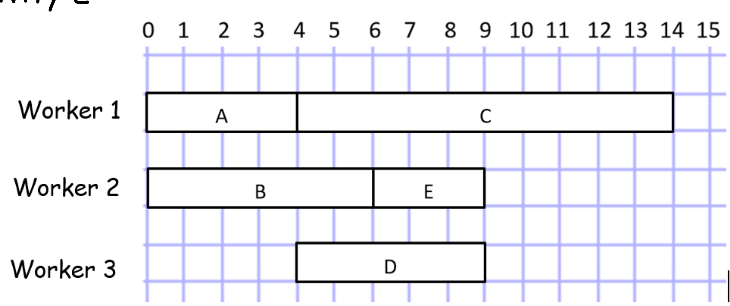
Step 1:

- We start by assigning Activity A to worker 1.
- Worker 2 is now idle, so we assign them the next activity with the lowest late time i.e. Activity B.
- Worker 3 is now idle, so we assign them the next activity with the lowest late time i.e. Activity D



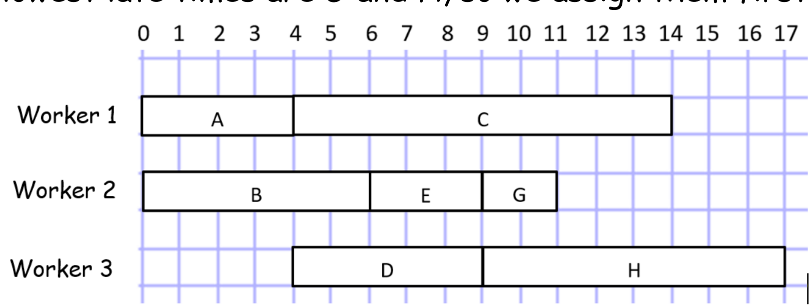
Step 2:

- Worker 1 is finished on Day 4, so the next available activity they can start, with lowest late time is Activity C.
- Worker 2 is free after Day 6, so the next available activity they can start, with lowest late time is Activity E



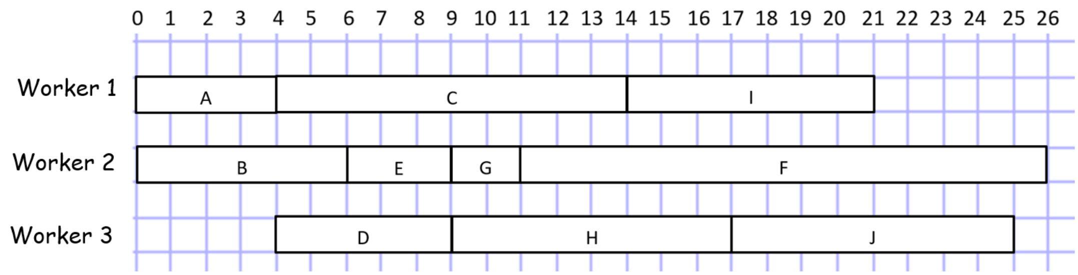
Step 3:

- Both workers 2 and 3 are now free on Day 9, so we can assign either of activities F, G or H but the two lowest late times are G and H, so we assign them first:



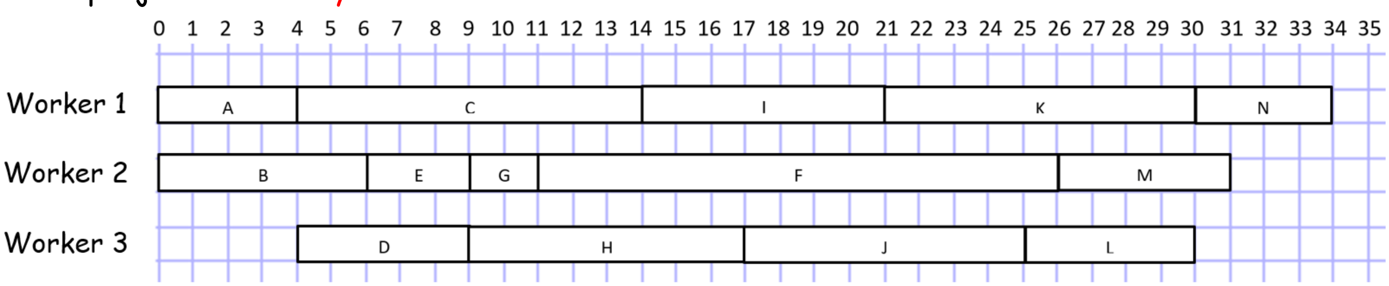
Step 4:

- Worker 2 is free after Day 11, so we can now assign them F, J or K.
- Activities F and J have the same late time, so we will assign F, as it is first in alphabetical order.
- Worker 1 is free after Day 14, so Activities I or J can be assigned next, so we follow alphabetical order again and assign I
- Worker 3 will be free after Day 17, so J has the lowest late time of activities ready to be assigned.



Step 5:

- Worker 1 is free after Day 21, so K and L have the same late time, so we assign K.
- Worker 3 is then available after Day 25, so we assign the next lowest late time of L to them.
- We are left with activities M and N, so we assign M to Worker 2 as they become free before the other 2 workers.
- Finally, we can assign N to either Worker 1 or 3 to bring the completion time for the project to 34 days.



Classwork Questions: Pg 270/271 Ex 12G Qs 1/4/5/6/7

Topic 55: Bellman's Principle of Optimality

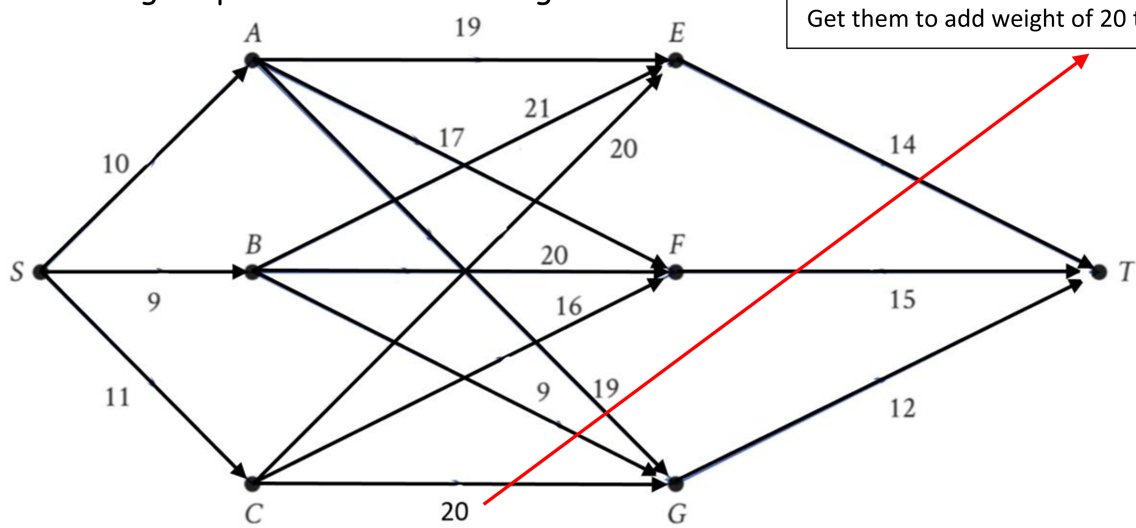
- See Handout 12 – 14 and Bellman's Algorithm

- Example: Pg 276/277 Ex 12H Q2

(i) Using the table provided, find the longest route from S to T in this multi-stage network

(ii) Name the longest path and state its length.

Get them to add weight of 20 to [CG]

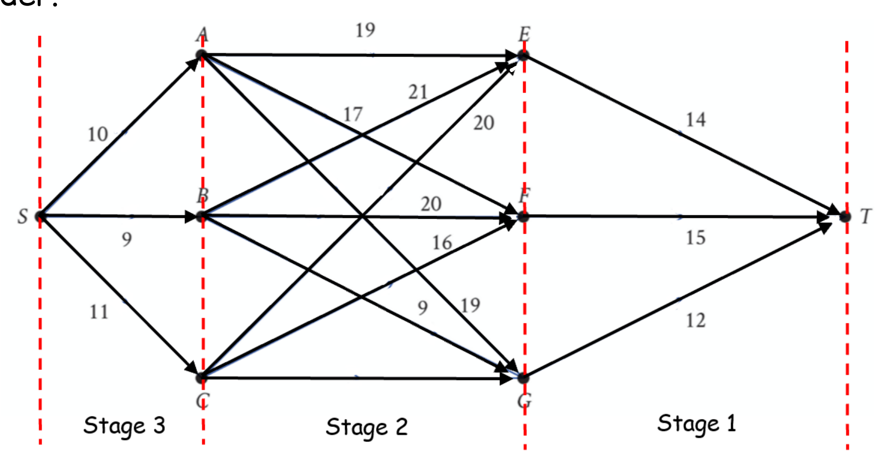


Stage	State	Action	Value
1	E	ET	
	F	FT	
	G	GT	
2	A	AE	
		AF	
		AG	
	B	BE	
		BF	

Stage	State	Action	Value
		BG	
	C	CE	
		CF	
		CG	
3	S	SA	
		SB	
		SC	

Solution:

- The first step is to mark in the stages in the diagram. Note that the stages are numbered in reverse order.



- Now we can fill in all the weights into the table, marking the optimum ones with asterisks. In this question, we are looking for the longest route from S to T, so "optimum" means the longest route from S to T.

Stage	State	Action	Value	Explanation
1	E	ET	14*	Only 1 route from E to T so 14 is optimal
	F	FT	15*	Only 1 route from F to T so 15 is optimal
	G	GT	12*	Only 1 route from G to T so 12 is optimal
2	A	AE	33*	Adding Edge AE (19) to the optimal value of E i.e. 14 As 33 is the biggest of 31, 32, 33, this is the optimal
		AF	32	Adding Edge AF (17) to the optimal value of F i.e. 15
		AG	31	Adding Edge AG (19) to the optimal value of G i.e. 12
	B	BE	35	Adding Edge BE (21) to the optimal value of E i.e. 14
		BF	35*	Adding Edge BF (20) to the optimal value of F i.e. 15 As 35 is the biggest of 31, 35, 21, this is the optimal
		BG	21	Adding Edge BG (9) to the optimal value of G i.e. 12
	C	CE	34*	Adding Edge CE (20) to the optimal value of E i.e. 14 As 34 is the biggest of 34, 31, 32, this is the optimal
		CF	31	Adding Edge CF (16) to the optimal value of F i.e. 15
		CG	32	Adding Edge CG (20) to the optimal value of G i.e. 12
3	S	SA	43	Adding Edge SA (10) to the optimal value of A i.e. 33
		SB	44	Adding Edge SB (9) to the optimal value of B i.e. 35
		SC	45*	Adding Edge SC (11) to the optimal value of C i.e. 34 As 45 is the biggest of 43, 44, 45, this is the optimal

- Looking at the asterisked values now, we can see that the longest route from source to sink is SCET, which has length of 45.

Classwork Questions: Pg Ex 12H Qs 1/3/5/7/8

- Topic 56: Dynamic Programming

  a) Routing Problems:

- Example 1:

Sam has an ice-cream van from which she sells ice cream. She visits a different festival every week. She has to decide which three festivals to visit over the next three weeks. She starts the three-week period at home and finishes at home. She will spend one week at each of the three festivals she chooses, by travelling directly from one festival to the next. Table 1 gives the week in which 8 possible festivals will be held. Table 2 gives the expected profits from visiting each festival. Table 3 gives the cost of travel between festivals (and between home and festivals).

Table 1:

Week	1	2	3
Festivals	A,B,C	D,E	F,G,H

Table 2:

Festival	A	B	C	D	E	F	G	H
Expected Profit	900	800	1000	1500	1300	500	700	600

Table 3:

Travel Cost (€)	A	B	C	D	E	F	G	H
Home	100	80	150			80	90	70
A				180	150			
B				140	120			
C				200	210			
D						200	160	120
E						170	100	110

(i) Use Dynamic Programming to find the schedule that maximises the total expected profit, taking into account travel costs.

(ii) Which festivals should she visit and what is the maximum expected profit?

Solution:

- We work backwards in these problems, so we start with the final week.
- Sam could be at festival F, G or H and drives home at the end of the week.

Stage	State	Action	Destination	Value
# weeks to go	Where Sam visits	Where she travels	Where she finishes	= Profit – Travel + OVD
1	F	F -> Home	Home	500 – 80 + 0 = 420*
	G	G -> Home	Home	700 – 90 + 0 = 610*
	H	H -> Home	Home	600 – 70 + 0 = 530*

OVD = Optimum Value from Destination to sink

- We now go back to the previous week, when she could have been at festivals D or E.

Stage	State	Action	Destination	Value
# weeks to go	Where Sam visits	Where she travels	Where she finishes	= Profit – Travel + OVD
1	F	F -> Home	Home	500 – 80 + 0 = 420*
	G	G -> Home	Home	700 – 90 + 0 = 610*
	H	H -> Home	Home	600 – 70 + 0 = 530*
2	D	DF	F	1500 – 200 + 420 = 1720
		DG	G	1500 – 160 + 610 = 1950*
		DH	H	1500 – 120 + 530 = 1910
	E	EF	F	1300 – 170 + 420 = 1550
		EG	G	1300 – 100 + 610 = 1810*
		EH	H	1300 – 110 + 530 = 1720

- Finally, we go back to the first week:

Stage	State	Action	Destination	Value
# weeks to go	Where Sam visits	Where she travels	Where she finishes	= Profit – Travel + OVD
1	F	F -> Home	Home	500 – 80 + 0 = 420*
	G	G -> Home	Home	700 – 90 + 0 = 610*
	H	H -> Home	Home	600 – 70 + 0 = 530*
2	D	DF	F	1500 – 200 + 420 = 1720*
		DG	G	1500 – 160 + 610 = 1950*
		DH	H	1500 – 120 + 530 = 1910
	E	EF	F	1300 – 170 + 420 = 1550
		EG	G	1300 – 100 + 610 = 1810*
		EH	H	1300 – 110 + 530 = 1720
3	A	AD	D	900 – 180 + 1950 = 2670*
		AE	E	900 – 150 + 1810 = 2560
	B	BD	D	800 – 140 + 1950 = 2610*
		BE	E	800 – 120 + 1810 = 2490
	C	CD	D	1000 – 200 + 1950 = 2750*
		CE	E	1000 – 210 + 1810 = 2600
4	Home	Home -> A	A	-100 + 2670 = 2570
		Home -> B	B	-80 + 2610 = 2530
		Home -> C	C	-150 + 2750 = 2600*

(i) The optimal paths following the asterisks is Home – C – D – G – Home

(ii) She should visit the festivals C, D and G and will make an overall profit of €2600.

Classwork Questions: Pg 286 Ex 12I Qs 1 - 3

b) Stock Control Problems:

- In these types of problems:

  Stage = Time period in question

  State = Number of items in stock

  Action = Number of items made

  Destination = Number of items left in stock for the next time

- Example 2:

Alex produces electric scooters. He can produce up to four a month, but if he wishes to produce more than three in any month, he will have to hire an assistant at a cost of €350 for that month. In any month when scooters are produced, the overheads (heating, electricity, lighting etc.) cost €200. A maximum of three scooters can be held in stock in any one month, at a cost of €40 per scooter per month. Scooters are delivered at the end of the month. The order book for scooters (all of which must be met on time) is:

Month	Aug	Sep	Oct	Nov
No. of Scooters Required	3	3	5	2

(i) What is the cost of storing 2 scooters and producing 4 scooters in a given month?

(ii) If there is no stock at the beginning of August and Alex plans to have no stock in storage at the end of November, find the minimum cost for Alex to meet his orders and how he does it.

(iii) If the parts are imported from China at a cost of €600 per scooter and if Alex pays himself a salary of €1800 per month, and if he sells the scooters of €2000 each, find the overall profit over the 4-month period.

Solution:

(i) Cost = Storage + Overheads + Hired Labour

= (2 x 40) + 200 + 350

= €630

(ii) Again, we start from the last month first:

Stage (Demand)	State	Action	Destination	Value
The month (No. in order book)	No. in Stock	No. Made	No. Left in Stock	Costs = Storage + Overheads + Hired Labour + OVD
Nov(2)	0	2	0	0 + 200 + 0 = 200*
	1	1	0	40 + 200 + 0 = 240*
	2	0	0	80 + 0 + 0 = 80*
	3	0	1	This cannot happen as we want to clear our stocks by the end of Nov

OVD = Optimal Value from the Destination

- Now, working back to October, there is a demand for 5 new scooters.
- Alex can only make 4 (even with hired help), so we need to have at least 1 scooter in storage at the beginning of October.

Stage (Demand)	State	Action	Destination	Value
The month (No. in order book)	No. in Stock	No. Made	No. Left in Stock	Costs = Storage + Overheads + Hired Labour + OVD
Nov(2)	0	2	0	0 + 200 + 0 = 200*
	1	1	0	40 + 200 + 0 = 240*
	2	0	0	80 + 0 + 0 = 80*
	3	0	1	This cannot happen as we want to clear our stocks by the end of Nov
Oct (5)	1	4	0	40 + 200 + 350 + 200 = 790*
	2	3	0	80 + 200 + 0 + 200 = 480*
		4	1	80 + 200 + 350 + 240 = 870
	3	2	0	120 + 200 + 0 + 200 = 520*
		3	1	120 + 200 + 0 + 240 = 560
		4	2	120 + 200 + 350 + 80 = 750

- Going back a further month:

Stage (Demand)	State	Action	Destination	Value
The month (No. in order book)	No. in Stock	No. Made	No. Left in Stock	Costs = Storage + Overheads + Hired Labour + OVD
Nov(2)	0	2	0	0 + 200 + 0 = 200*
	1	1	0	40 + 200 + 0 = 240*
	2	0	0	80 + 0 + 0 = 80*
	3	0	1	This cannot happen as we want to clear our stocks by the end of Nov
Oct (5)	1	4	0	40 + 200 + 350 + 200 = 790*
	2	3	0	80 + 200 + 0 + 200 = 480*
		4	1	80 + 200 + 350 + 240 = 870
	3	2	0	120 + 200 + 0 + 200 = 520*
		3	1	120 + 200 + 0 + 240 = 560
		4	2	120 + 200 + 350 + 80 = 750
Sep (3)	0	3	0	Not allowed as we need to have at least 1 in stock next month
		4	1	0 + 200 + 350 + 790 = 1340*
	1	3	1	40 + 200 + 0 + 790 = 1030*
		4	2	40 + 200 + 350 + 480 = 1070
	2			Not possible to have 2 in stock (as would have had to make 5 in August)

- And finally, back to August:

Stage (Demand)	State	Action	Destination	Value
The month (No. in order book)	No. in Stock	No. Made	No. Left in Stock	Costs = Storage + Overheads + Hired Labour + OVD
Nov(2)	0	2	0	0 + 200 + 0 = 200*
	1	1	0	40 + 200 + 0 = 240*
	2	0	0	80 + 0 + 0 = 80*
	3	0	1	This cannot happen as we want to clear our stocks by the end of Nov
Oct (5)	1	4	0	40 + 200 + 350 + 200 = 790*
	2	3	0	80 + 200 + 0 + 200 = 480*
		4	1	80 + 200 + 350 + 240 = 870
	3	2	0	120 + 200 + 0 + 200 = 520*
		3	1	120 + 200 + 0 + 240 = 560
		4	2	120 + 200 + 350 + 80 = 750
Sep (3)	0	3	0	Not allowed as we need to have at least 1 in stock next month
		4	1	0 + 200 + 350 + 790 = 1340*
	1	3	1	40 + 200 + 0 + 790 = 1030*
		4	2	40 + 200 + 350 + 480 = 1070
	2			Not possible to have 2 in stock (as would have had to make 5 in August)
Aug (3)	0	3	0	0 + 200 + 0 + 1340 = 1540*
	0	4	1	0 + 200 + 350 + 1030 = 1580

(ii) Alex's optimum costs are €1540 if he makes 3 scooters in August, 4 in September, 4 in October and 2 in November.

(iii) Overall Profit = Profit on 13 Scooters – Salary - Costs

= 13(2000 – 600) – 4(1800) – 1540

= €9460

Classwork Questions: Pg 287 Ex 12I Qs 4 - 6

c) Allocation of Resources:

- In these problems, we are looking to allocate a finite amount of resources between a number of uses.
- The resources allocated to one use, will depend on the resources allocated to another.
- The order doesn't matter for this type, but we still work from sink to source.

- Example 3:

Sarah owns an apple orchard. Every year after the harvest, she sells the apples to a supermarket chain. If she has a particularly good crop, she uses the excess apples to make either Apple Sauce, Cider or feed for pigs. This year she has 5 tonnes left after delivering her orders to the supermarket chain. The price (in euro) she will get for these is as follows:

Tonnes	1	2	3	4	5
Feed for Pigs	85	165	245	325	405
Cider	165	185	205	225	245
Apple Sauce	125	275	335	365	385

Sarah will allocate her 5 tonnes in units of 1 tonne. She has already got an order for Apple Sauce, which will use 1 tonne of apples. (This means that she can use at most 4 tonnes for any other commodity)

(i) Which would get more profit: allocating 2 tonnes each to pig-feed and cider, along with one tonne to apple sauce OR 2 tonnes to cider and 3 to apple sauce?

(ii) Use dynamic programming to decide how Sarah should allocate her 5 tonnes of surplus apples in order to optimize her profits – and what is the maximum profit?

Solution:

(i) First option = 165 + 185 + 125 = €475. Second option = 185 + 335 = €520. The second option makes more profit.

(ii)

Stage	State	Action	Destination	Value
(Product)	(Tonnes available)	(Tonnes allocated)	(Tonnes remaining)	= profit + OVD
3	1	1	0	125 + 0 = 125*
(apple sauce)	2	2	0	275 + 0 = 275*
	3	3	0	335 + 0 = 335*
	4	4	0	365 + 0 = 365*
	5	5	0	385 + 0 = 385*
2	0	0	0	Cannot happen as we need to leave at least 1 tonne for A (Apple sauce)

(Cider)	1	0	1	0 + 125 = 125*
	2	0	2	0 + 275 = 275
		1	1	165 + 125 = 290*
	3	0	3	0 + 335 = 335
		1	2	165 + 275 = 440*
		2	1	185 + 125 = 310
	4	0	4	0 + 365 = 365
		1	3	165 + 335 = 400
		2	2	185 + 275 = 460*
		3	1	205 + 125 = 330
	5	0	5	0 + 385 = 385
		1	4	165 + 365 = 530*
		2	3	185 + 335 = 520
		3	2	205 + 125 = 330
		4	1	225 + 125 = 450
1	5	5	0	Impossible (must have at least 1 tonne)
(Pig Feed)		4	1	325 + 125 = 550
		3	2	245 + 290 = 535
		2	3	165 + 440 = 605*
		1	4	85 + 460 = 545
		0	5	0 + 530 = 530

- The optimal plan is to use 2 tonnes of apples for pig-feed, 1 tonne for cider and 2 for apple sauce.
- This will give her a profit of €605.

Classwork Questions: Pg 288 Ex 12I Qs 7 - 9

d) Equipment Replacement and Maintenance:

- Example 4:

A new van costs €25,000. Maintenance costs are as follows: €300 in the first year, €500 in the second year and €1000 in the third year (as tyres, batteries and brakes begin to wear). The resale value of a second-hand van is €20,000 after 1 year, €17,000 after 2 years and €15,000 after 3 years. A self-employed house painter buys a new van and wants to use dynamic programming to decide what is the best strategy over the next 6 years: how often should she replace her van over these 6 years? Assume that there is no re-sale value after 3 years (she must sell after 1, 2 or 3 years).

Solution:

- In this question, the value will be the cost:

= replacement cost + maintenance cost – resale value + OVD

- We start by looking at the last 2 years:

Stage	State	Action	Destination	Value (Cost)
(Year)	(No. of yrs left when you buy this van)	(Decide to keep van for his no. of years)	(No. of yrs left when you sell the van)	= Cost of a new van + maintenance – resale value + OVD
0	0	0	0	0*
1	1	1	0	25000 + 300 – 20000 = 5300*
2	2	2	0	25000 + 800 – 17000 + 0 = 8800*
		1	1	25000 + 300 – 20000 + 5300 = 10600

- Working backwards as always and after year 6, the whole project stops, and we stop.
- In year 4, she can choose between keeping the van 2 years (and selling at the end) or keeping the van for 1 year and selling with 1 year to go.
- As the table shows, the optimal one of these (with minimum cost) is the former, at a cost of €8800.
- The cost of maintaining the van for the first 2 years is €300 + €500 = €800.

- So, continuing to work back:

Stage	State	Action	Destination	Value (Cost)
(Year)	(No. of yrs left when you buy this van)	(Decide to keep van for this no. of years)	(No. of yrs left when you sell the van)	= Cost of a new van + maintenance – resale value + OVD
0	0	0	0	0*
1	1	1	0	25000 + 300 – 20000 = 5300*
2	2	2	0	25000 + 800 – 17000 + 0 = 8800*
		1	1	25000 + 300 – 20000 + 5300 = 10600
3	3	3	0	25000 + 1800 – 15000 + 0 = 11800*
		2	1	25000 + 800 – 17000 + 5300 = 14100
		1	2	25000 + 300 – 20000 + 8800 = 14100
4	4	3	1	25000 + 1800 – 15000 + 5300 = 17100*
		2	2	25000 + 800 – 17000 + 8800 = 17600
		1	3	25000 + 300 – 20000 + 11800 = 17100*
5	5	3	2	25000 + 1800 – 15000 + 8800 = 20600*
		2	3	25000 + 800 – 17000 + 11800 = 20600*
		1	4	25000 + 300 – 20000 + 17100 = 22400
6	6	3	3	25000 + 1800 – 15000 + 11800 = 23600*
		2	4	25000 + 800 – 17000 + 17100 = 25900
		1	5	25000 + 300 – 20000 + 20600 = 25900

- The optimal path (following asterisks) is to sell after 3 years and at the end.

Classwork Questions: Pg Ex 12I Qs 10 - 12
Revision Questions and Test