Leaving Certificate Examination

Sample Paper 2

Applied Mathematics

Higher Level

2 hours and 30 minutes

400 marks

Examination Number

For examiner	
Question	Mark
1	/50
2	/50
3	/50
4	/50
5	/50
6	/50
7	/50
8	/50
~~9~~	~~/50~~
~~10~~	~~/50~~
Written Total	/400
Project	/100
Overall Total	/500
Overall Grade	

Sample Paper 2

Question 1

(a)

A parcel rests on the horizontal floor of a van.
The van is travelling on a level road at 14 m s^{-1} .
It is brought to rest by a uniform application of the brakes.
The coefficient of friction between the parcel and the floor is $\frac{2}{5}$.
Show that the parcel is on the point of sliding forward on the floor of the van if the stopping distance is 25 m.

(b)

A particle, of mass m falls vertically downwards under gravity.
At time t, the particle has speed v and it experiences a resistance force of magnitude kmv, where k is a constant.
The initial speed of the particle is u.

(i) Show that $v = \frac{g}{k} - \left(\frac{g}{k} - u\right)e^{-kt}$, at time t.

(ii) If $u = 9{\cdot}8 \text{ m s}^{-1}$ and $k = 0{\cdot}98 \text{ s}^{-1}$, find the distance travelled by the particle in 4 seconds.

(Note: $\int \frac{dx}{a+bx} = \frac{1}{b}\ln|a + bx| + c$).

Question 2

(a)

Beatrix is going to college next year. She buys a new laptop for her studies. She will be in college for 4 years. The laptop costs €2000. She will sell whatever laptop she has at the end of the 4 years. The replacement value for this laptop each year and the maintenance costs are shown in the tables below:

Years old	1	2	3	4
Value (€)	1400	900	400	100

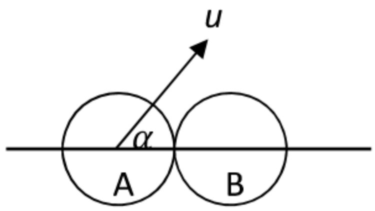
Years	1	2	3	4
Maintenance Cost(€)	50	200	300	350

(i) If Beatrix replaces her laptop after 2 years and sells again after 4 years, what will her costs amount to?

(ii) Use dynamic programming to find the minimum possible costs and the strategy which gives rise to it.

(b)

A smooth sphere, A, of mass m collides obliquely with another smooth sphere, B, of mass m.



Before impact, A is moving with speed u at an angle α to the line of centres of the spheres, where $0° < \alpha < 45°$.

B is at rest before the impact.

The coefficient of restitution for the collision is e.

(i) Find the speed of A and the speed of B after impact in terms of u, e and α.

(ii) Given that A is deflected through angle α because of the collision, show that $\tan^2 \alpha = e$.

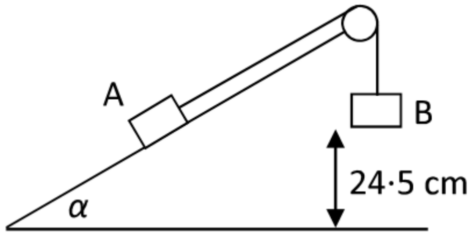
Question 3

(a)

The acceleration of a particle (in ms^{-2}) is determined by the equation $a = v^2 + 25\ m/s^2$. The initial speed of the particle is 0. Find the distance travelled by the particle as its speed increases from $1\ m/s$ to $4\ m/s$, to 3 significant figures.

(b)

A block A of mass $10m$ on a smooth plane inclined at an angle α with the horizontal, where $\tan \alpha = \frac{3}{4}$, is connected by a light inextensible string which passes over a smooth pulley to a second block B of mass $10m$. B is 24·5 cm above an inelastic horizontal floor, as shown in the diagram.



The system is released from rest.

Find

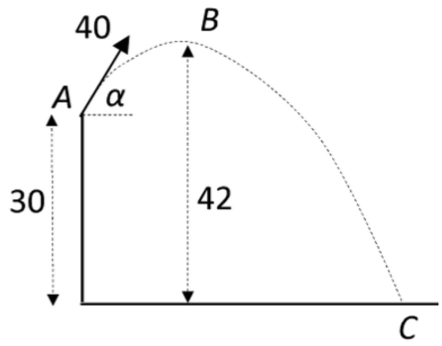
(i) the acceleration of B

(ii) the time that B remains in contact with the floor.

Question 4

(a)

A particle is projected with speed 40 m s^{-1} from a point *A* on the top of a vertical cliff of height 30 m.
The maximum height reached by the particle is 42 m above the horizontal ground, at point *B*.
It strikes the ground at *C*.



Find

(i) the value of α, the angle of projection

(ii) the horizontal range of the particle

(iii) the speed of the particle as it hits the ground at *C*.

(b)

A particle is projected horizontally along a smooth horizontal surface with initial speed 80 m s^{-1}. The particle has a retardation of $\frac{v}{100}$ m s^{-2}, where v is the speed.

Find

(i) the speed of the particle after *t* seconds

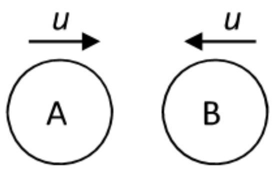
(ii) the distance travelled in t seconds

(iii) the speed v in terms of the distance travelled, s.

Question 5

(a)

A smooth sphere A of mass $4m$, moving with speed u on a smooth horizontal table collides directly with a smooth sphere B of mass m, moving in the opposite direction with speed u.



The coefficient of restitution between A and B is e.

(i) Find the speed, in terms of u and e, of each sphere after the collision.

The magnitude of the impulse on B due to the collision is T.

(ii) Show that $\frac{8mu}{5} \leq T \leq \frac{16mu}{5}$.

(b)

In a certain state in America, the population of pheasants is 25,000. The gun club release 3000 pheasant chicks into the wild every spring. The chances of a pheasant surviving through the shooting season and into the next year is 0.15.

(i) If P_n = the pheasant population in the state after n years, write down a difference equation which describes this situation.

(ii) Given that $P_0 = 25000$ find P_n in terms of n.

(iii) Estimate the pheasant population after 3 years.

(iv) Show that P_n approaches a steady state as the years go on and find that steady state.

Question 6

(a)

A particle of mass m is moving in such a way that its displacement (in metres) at time t (in seconds) from a fixed point O is given by

$$\vec{s} = (r\cos\omega t)\vec{\imath} + (r\sin\omega t)\vec{\jmath}$$

(i) Show that the magnitude of its displacement from O is a constant r.

(ii) Find the acceleration vector at any time t.

(iii) Show that the force exerted on the particle is directed towards O and is of magnitude $m\omega^2 r$.

(b)

Car C, moving with uniform acceleration f passes a point P with speed u (> 0). Two seconds later car D, moving in the same direction with uniform acceleration $2f$ passes P with speed $\frac{6}{5}u$. C and D pass a point Q together. The speeds of C and D at Q are 6.5 m s^{-1} and 9 m s^{-1} respectively.

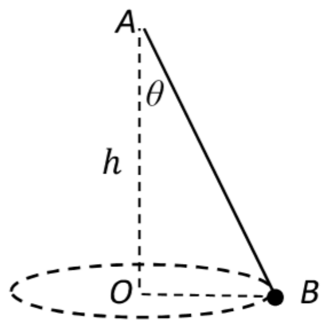
(i) Show that C travels from P to Q in $(\frac{3}{2f} + 5)$ seconds.

(ii) Find the value of f.

Question 7

(a)

One end A of a light elastic string is attached to a fixed point. The other end, B, of the string is attached to a particle of mass m. The particle moves on a smooth horizontal table in a circle with centre O, where O is vertically below A and $|AO| = h$. The string makes an angle θ with the downward vertical and B moves with constant angular speed ω about OA.



(i) Show that $\omega^2 \leq \frac{g}{h}$.

The elastic string has natural length h and elastic constant $\frac{2mg}{h}$.

(ii) Given that $\omega^2 = \frac{2g}{5h}$, find the value of θ.

(b)

A particle is projected vertically upwards with a velocity of u m s^{-1}.
After an interval of $2t$ seconds a second particle is projected vertically upwards from the same point and with the same initial velocity.

They meet at a height of h m.

Show that $h = \dfrac{u^2 - g^2t^2}{2g}$.

Question 8

(a)

One method of dyeing a piece of cloth is to immerse it in a container which has *P* grams of dye dissolved in a fixed volume of water.

The cloth absorbs the dye at a rate proportional to the mass of dye remaining.

$$\frac{dx}{dt} = k(P - x)$$

where *t* is time in seconds, *x* is the mass of dye absorbed by the cloth and $k = \frac{1}{50}$.

(i) Find the time taken to dye a piece of cloth if a mass of $\frac{5}{8}P$ needs to be absorbed to reach the desired colour.

(Note: $\int \frac{dx}{a+bx} = \frac{1}{b}\ln|a + bx| + c$)

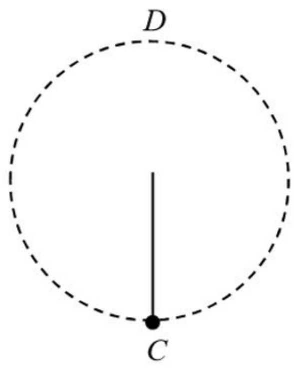
An alternative method is to keep the mass of dye present in the water constant at P grams by continuously adding dye throughout the process.

(ii) Find the time taken to dye the piece of cloth to the desired colour using this method.

(b)

A small particle hanging on the end of a light inextensible string 2 m long is projected horizontally from the point C.

(i) Calculate the least speed of projection needed to ensure that the particle reaches the point D which is vertically above C.



(ii) If the speed of projection is 7 m s^{-1} find the angle that the string makes with the vertical when it goes slack.

Page for extra work.

Label any extra work clearly with the question number and part.

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