Leaving Certificate Examination

Sample Paper 4

Applied Mathematics

Higher Level

2 hours and 30 minutes

400 marks

Examination Number

For examiner	
Question	Mark
1	/50
2	/50
3	/50
4	/50
5	/50
6	/50
7	/50
8	/50
~~9~~	~~/50~~
~~10~~	~~/50~~
Written Total	/400
Project	/100
Overall Total	/500
Overall Grade	

Sample Paper 4

Question 1

(a)

A smooth sphere, P, of mass $3m$ collides directly with another smooth sphere, Q, of mass $5m$. P and Q are moving in opposite directions before impact with speeds $4u$ and $2u$ respectively. The coefficient of restitution for the collision is e.

(i) Find the speed of P and the speed of Q after impact in terms of u and e.

(ii) If P and Q are moving in the same direction after impact, show that $0 \leq e < \frac{1}{15}$.

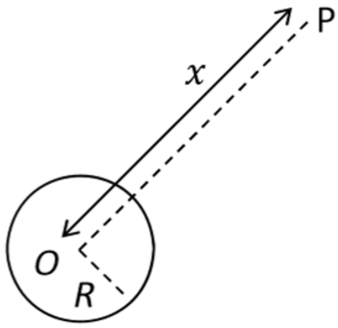
(b)

Evaluate $\int \sin^{-1} 2x \, . \, dx$.

Question 2

(a)

A spacecraft P of mass m moves in a straight line towards O, the centre of the earth.
The radius of the earth is R.
When P is a distance x from O, the force exerted by the earth on P is directed towards O and has magnitude $\frac{k}{x^2}$, where k is a constant.



(i) Show that $k = mgR^2$.

P starts from rest when its distance from O is $5R$.

(ii) Find, in terms of R, the speed of P as it hits the surface of the earth, given that air resistance can be ignored.

(b)

The population of ladybirds in a certain European country is measured in billions. The Entomological Society of the country started estimating the population last year. They estimate that the original population (in billions) was $P_0 = 25$. A year later, they estimate that $P_1 = 16$. The Entomological Society has stated that the population P_n in the n^{th} year is given by the difference equation

$$P_{n+2} = \frac{1}{5}(P_{n+1} + 4P_n)$$

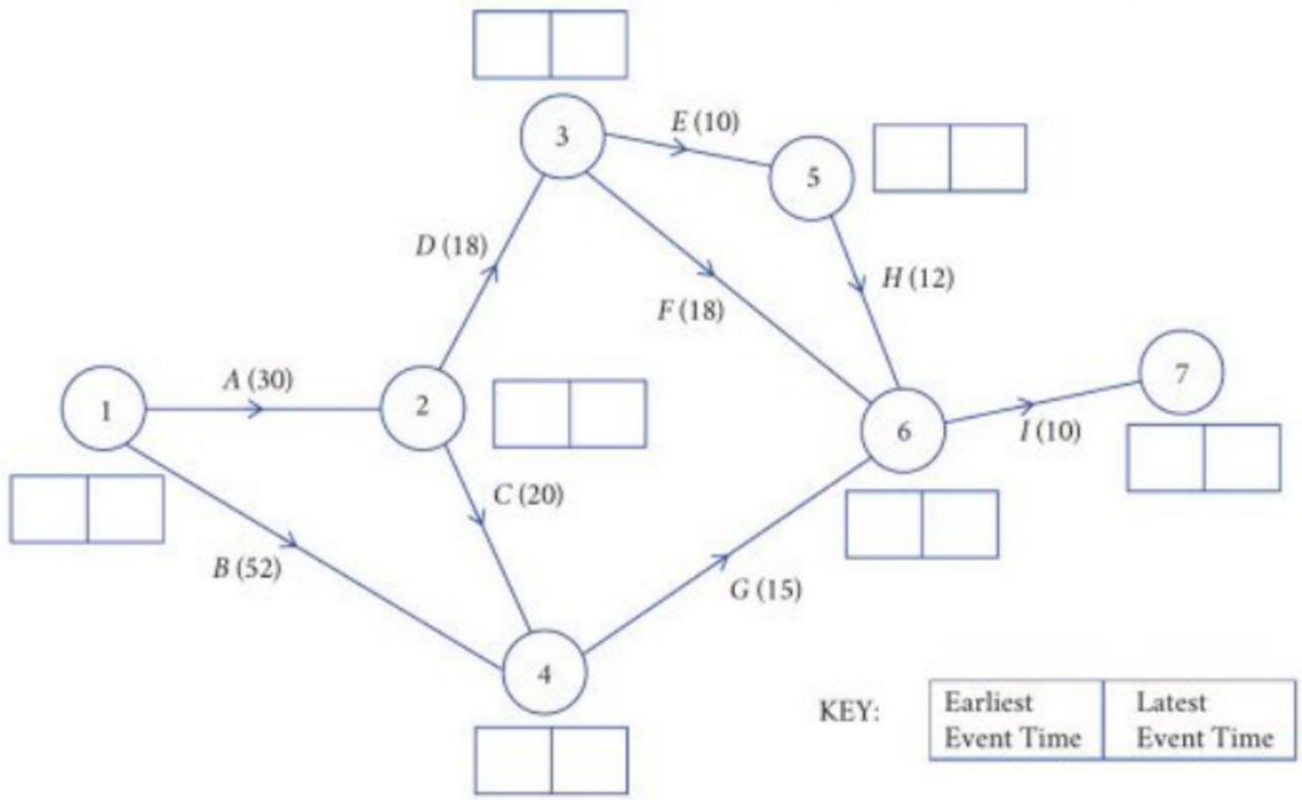
(i) Solve the difference equation.

(ii) Estimate the population in billions over the next two years i.e. P_2 and P_3

(iii) Draw a graph showing the population over the first four years.

(iv) As the years go on, the population reaches a steady state. Find this steady state value.

Question 3
(a)
A building project is modelled mathematically by the activity network shown below. The activities in the project are represented by the arcs. The numbers in brackets on each arc gives the time (in days) taken to complete the activity.



(i) Complete the diagram above with the early and late times for the project.
(ii) Write down the critical activities and the length of the critical path.

(iii) Calculate the total float for any non-critical activity in the network.

(iv) Draw up a schedule to determine the minimum number of workers needed to complete the project.

(b)

An object falls vertically, from rest, from a height *h* metres. It travels $\frac{15}{64}h$ metres during its final second of motion before hitting the ground.

(i) Find the time it takes to fall to the ground.

(ii) Find the value of *h*.

Question 4

(a)

At time t seconds the acceleration a m s^{-2} of a particle, P, is given by

$$a = 8t + 4.$$

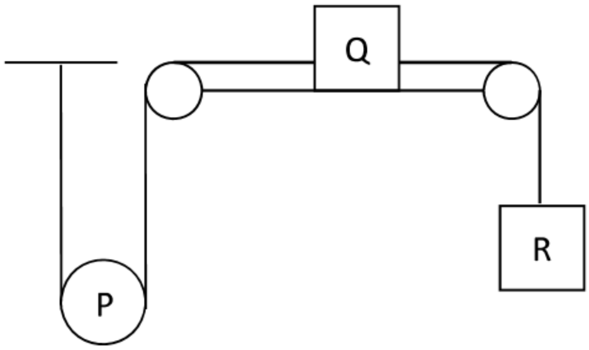
At $t = 0$, P passes through a fixed point with velocity -24 m s^{-1}.

(i) Show that P changes its direction of motion only once in the subsequent motion.

(ii) Find the distance travelled by P between $t = 0$ and $t = 3$.

(b)

A taut light inelastic string is fixed at one end and passes under a moveable pulley, P, of mass 4 kg which hangs vertically. The other end of the string is attached to Q, a mass of 4 kg which lies on a rough horizontal surface.



A second inelastic string connects Q to R, a mass of 10 kg which hangs vertically.

The fixed pulleys are smooth and light and the coefficient of friction between Q and the surface is $\frac{1}{2}$.

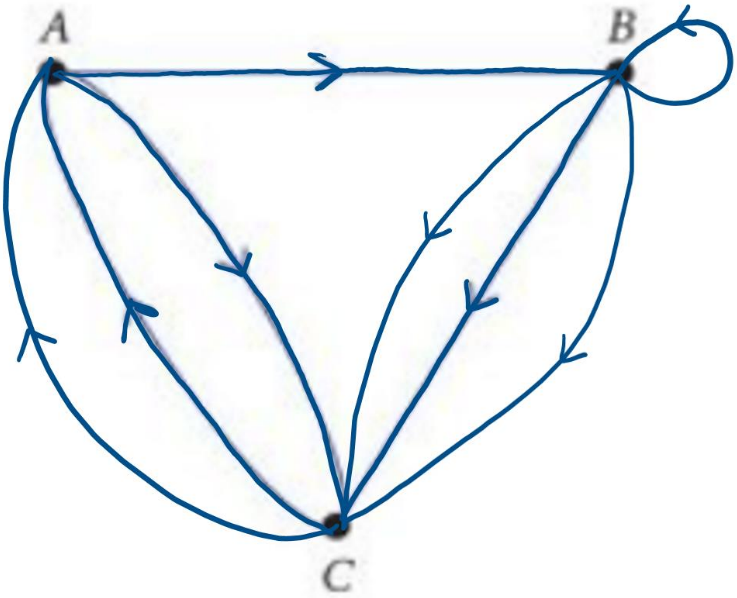
The system is released from rest.

Find the accelerations of P, Q and R in terms of g.

Question 5

(a)

(i) Write out the adjacency matrix M for this directed graph:S



(ii) Calcluate M^3.

(iii) How many walks of length 3 are there from A to C? Write down one of them.

(b)

A particle P is attached to one end of a light inextensible string of length d. The other end of the string is attached to a fixed point. The particle is hanging freely at rest, with the string vertical, when it is projected horizontally with speed u.
The particle moves in a complete vertical circle.

(i) Show that $u \geq \sqrt{5gd}$.

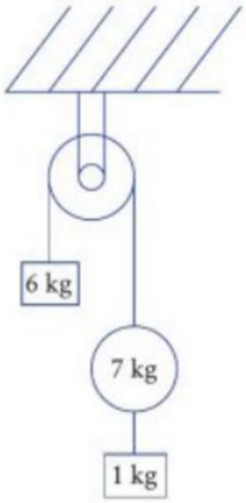
As P moves in the circle the least tension in the string is T_1 and the greatest tension is kT_1.

(ii) Given that $u = \sqrt{6gd}$, find the value of k.

Question 6

(a)

Two particles of mass 6 kg and 1 kg hang from a smooth pulley at the ends of a light inextensible string. The system is released from rest. After 1.5 seconds the 1 kg mass picks up a 7 kg mass. How much further will the 6 kg mass descend after this moment?



(b)

A new Biodiversity Park is opened in Paris. The number of visitors in the first year is 300,000. The following year the number of visitors goes up to 400,000. The number of visitors (in 1000s) in the n^{th} year (V_n) is estimated to be given by the difference equation:

$$V_{n+2} = \frac{8}{25}(5V_{n+1} - 2V_n) + 4n + 32$$

(i) Solve the difference equation using $V_0 = 300$ and $V_1 = 400$.

(ii) The chairman of the board at the park states "Visitor numbers will settle down at around one million as time goes by". Is this true? Justify your answer.

Question 7

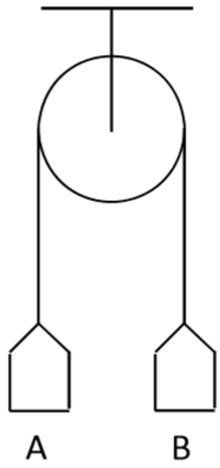
(a)

A car is travelling on a straight level road at a uniform speed of 26 m s^{-1} when the driver notices a tractor 91·2 m ahead.
The tractor is travelling at a uniform speed of 6 m s^{-1} in the same direction as the car.
The driver of the car hesitates for t seconds before applying the brake.
The maximum deceleration of the car is 5 m s^{-2}.

Find the maximum value of t which would avoid a collision between the car and the tractor.

(b)

Two scale pans A and B, each of mass *m* kg, are attached to the ends of a light inextensible string which passes over a light smooth fixed pulley. They are held at the same level, as shown in the diagram.
A mass of 3*m* kg is now placed on A.



The system is released from rest.

Find

(i) the tension in the string in terms of *m*

(ii) how far B has risen when it reaches a speed of $0{\cdot}4\ \text{m s}^{-1}$

(iii) the reaction on the 3*m* kg mass in terms of *m*.

Question 8

(a)

A particle is projected out to sea from a point *P* on a cliff to hit a target 60 m horizontally from *P* and 60 m vertically below *P*.

The velocity of projection is $14\sqrt{3}$ m s^{-1} at an angle α to the horizontal.

Find

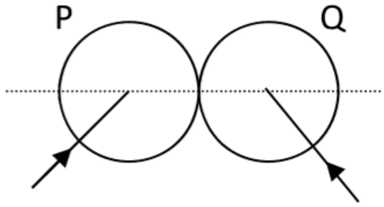
(i) the two possible values of α

(ii) the times of flight.

(b)

A smooth sphere P, of mass $2m$, collides with a smooth sphere Q, of mass m.
The velocity of P is $3u\,\vec{\imath} + 4u\,\vec{\jmath}$ and the velocity of Q is $-4u\,\vec{\imath} + 3u\,\vec{\jmath}$, where $\vec{\imath}$ is along the line of centres at impact.

The coefficient of restitution between the spheres is $\frac{5}{7}$.



Find

(i) in terms of u, the speed of each sphere after the collision

(ii) the angle between the directions of P and Q after the collision.

Page for extra work.

Label any extra work clearly with the question number and part.

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